Math 4050 Review #2

1. State the definition of an infinite decimal (in base 10), and prove that an infinite decimal must converge to a real number.

2. Prove: The terminating decimal $D.d_1 \ldots d_t$ (where $d_t > 0$) has a second decimal representation.

3. Convert the base-twelve duodecimal $3A.B5$ to a fraction, written in base 10.

4. Convert the base-sixteen hexadecimal $7.8\overline{AF}$ to a fraction, written in base 10.

5. Which of the following six assertions are true, and which are false? The lowest-terms fraction $\frac{m}{n}$ has a terminating decimal representation:

   - <u>if and only if</u> / <u>if</u> / <u>only if</u> n is divisible by no prime other than 2
   - <u>if and only if</u> / <u>if</u> / <u>only if</u> n is not divisible by 3

6. A rational number can be expressed as either a decimal, as a lowest-term fraction, and either in the form $\dfrac{M}{10^r}$ or $\dfrac{M}{10^r(10^l - 1)}$. Express each rational number below in the other two forms. Do as much work as you can without a calculator.

0.124	$0.\overline{386100}$	$0.0000\overline{27}$	$0.12\overline{345}$
$3/64$	$4/39$	$11/48$	$7/250$
$123/10000$	$3478/9999$	$49732/99999900$	$21497323/99999900$

7. Use the Division Algorithm and a calculator to find the decimal representation of 8/17.

8. Let $x = 4329218107/7792592592$. According to a calculator, $x = 0.555555556$. Does $x = 0.\overline{5}$?

9. Consider those reciprocals of primes that have simple-periodic decimal representations. Prove:

   - There is exactly one with period 1. What is it?
   - There is exactly one with period 2. What is it?
   - There is exactly one with period 3. What is it?
   - There is exactly one with period 4. What is it?

10. Consider those reciprocals of integers that have simple-periodic decimal representations. Prove:

    - There are exactly two with period 1. What are they?
    - There are exactly two with period 2. What are they?
    - There are exactly five with period 3. What are they?
    - There are exactly six with period 4. What are they?

11. Find a repeating, nonterminating decimal that begins as 0.222222222222222 (fifteen 2's) but is not equal to $0.\overline{2}$. Write your decimal as a fraction.

12. Prove that $x \in \mathbb{R} \setminus \mathbb{Q}$ if and only if x has an infinite nonrepeating decimal representation.

13. If $z \in \mathbb{C}$, prove that $|\mathrm{Re}(z)| \leq |z|$ and $|\mathrm{Im}(z)| \leq |z|$.

14. If $z \in \mathbb{C}$, prove that $|z| = |-z| = |\overline{z}|$.

15. If $z \in \mathbb{C}$, prove that $|z|^2 = z\overline{z}$.

16. If $z, w \in \mathbb{C}$, prove that $\overline{w+z} = \overline{w} + \overline{z}$.
17. If $z, w \in \mathbb{C}$, prove that $\overline{wz} = \overline{w}\overline{z}$.
18. If $a \in \mathbb{R}$ and $z \in \mathbb{C}$, prove that $\overline{az} = a\overline{z}$.
19. If $z, w \in \mathbb{C}$, prove that $|w+z| \leq |w| + |z|$.
20. If $z, w \in \mathbb{C}$, prove that $|wz| = |w||z|$.
21. State the division algorithm for polynomials.
22. Give a counterexample that shows the division algorithm is false for $\mathbb{Z}[x]$.
23. State and prove the Remainder Theorem for polynomials.
24. State and prove the Factor Theorem for polynomials.
25. Prove: For any odd positive integer n, $x + c$ is a factor of $x^n + c^n$.
26. Explain when and why synthetic division works.