Math 4050 **Practice Problem Set #10**

Problem 10.1 Find the equation of the tangent line to $y = \sqrt{x^2 + 9}$ at $x = 4$. Write your answer in slope-intercept form.

Problem 10.2 Differentiate

$$f(\theta) = \frac{\theta \sin^2 \theta^2}{1 + \sec 2\theta}$$

Problem 10.3 State the formal definition of a derivative. Use this formal definition to find the derivative of $f(x) = 5x - 3$. You *must* use the formal definition of a derivative to do this problem.

Problem 10.4 If possible, sketch the graph of a function so that f is continuous at $x = 2$ but $f'(2)$ does not exist. If this cannot be done, write the word "Impossible."

Problem 10.5 If possible, sketch the graph of a function so that $f'(2) = 0$ but f is discontinuous at $x = 2$. If this cannot be done, write the word "Impossible."

Problem 10.6 True or false:

- If $f(x)$ is continuous at a point c, then f must be differentiable at c.
- If $f(x)$ is differentiable at a point c, then f must be continuous at c.

Problem 10.7 If possible, sketch a function f which satisfy all of the following:

- $\lim_{x \to 4^+} f(x) = 3$
- $\lim_{x \to 4^-} f(x) = -2$
- $f(4) = 1$
- f is differentiable for all $x \neq 4$

If this is not possible, write the word "Impossible."

Problem 10.8 If possible, sketch a function f which satisfy all of the following:

- $\lim_{x \to \infty} f(x) = 3$
- $\lim_{x \to -\infty} f(x)$ is undefined but is neither ∞ or $-\infty$.
- f is differentiable for all x

If this is not possible, write the word "Impossible."

Problem 10.9 Find $\dfrac{dy}{dx}$ if

$$\sqrt{x^3 + y^2} = \cot\left(\frac{x}{y}\right).$$

Problem 10.10 Differentiate $y = \csc^2\left(x + \sqrt[3]{2x + 7}\right)$.

Problem 10.11 Find the values of x where $Q(x) = \dfrac{x}{x^2+1}$ has a horizontal tangent line.

Problem 10.12 Find $\dfrac{d^2y}{dx^2}$ if $y = \dfrac{1}{x^2+1}$.

Problem 10.13 Differentiate $y = x^2 \sin^6(\sec(x \tan 4x))$.

Problem 10.14 Differentiate

$$h(t) = \frac{\sqrt{t}}{4} - \frac{\sqrt[3]{t}}{5} + \frac{\sqrt[6]{t}}{10}.$$

Problem 10.15 Find the values of x where the slope of the tangent line to

$$f(x) = \frac{x}{2x+1}$$

has slope 1.