

Math 4050

Practice Problem Set #6

Problem 6.1 Prove the trigonometric identity

$$8 \sin^2 \theta \cos^2 \theta = 1 - \cos 4\theta$$

Problem 6.2 Simplify

$$\cos \left(\cos^{-1} x - \tan^{-1} \left(\frac{2}{3} \right) \right)$$

Problem 6.3 Prove the trigonometric identity

$$\frac{\sin x}{\cot x} = \sec x - \cos x$$

Problem 6.4 Use a calculator to compute

$$\sin 20^\circ \sin 40^\circ \sin 80^\circ.$$

Then prove this result using trigonometric identities.

Problem 6.5 Find the exact value of

$$\sin \left(\frac{17\pi}{24} \right) \sin \left(\frac{\pi}{24} \right)$$

Problem 6.6 Use the substitution $u = 3 \tan \theta$ to simplify

$$\frac{6u^2}{(9 + u^2)^{3/2}}$$

Your final answer should contain no fractions.

Problem 6.7 Prove the trigonometric identity

$$\frac{\cos 3x - \cos 7x}{\sin 3x + \sin 7x} = \frac{2 \tan x}{1 - \tan^2 x}$$

Problem 6.8 Suppose that $0 < x < 2\pi$, $\sin x = -\frac{5}{13}$, and $\cos x > 0$. Find $\cos \left(\frac{x}{2} \right)$ exactly.

Problem 6.9 Prove the trigonometric identity

$$\sin 3x = (1 + 2 \cos 2x) \sin x$$

Problem 6.10 Suppose that $\pi < x < 2\pi$ and $\cos x = \frac{7}{25}$. Use half-angle identities twice to find $\sin \left(\frac{x}{4} \right)$ exactly.

Problem 6.11 Prove the trigonometric identity

$$(1 - \cos 2x)(1 + \cos 2x) = \frac{1}{\csc^2(2x)}$$

Problem 6.12 Exactly evaluate

$$\cos \left(2 \tan^{-1}(-3) \right)$$

Problem 6.13 Determine k and ϕ so that

$$-2 \sin 2x + 3 \cos 2x = k \sin(2x + \phi).$$

Express ϕ in degrees, accurate to one decimal place.