

Review Problem 2.1 A random variable X has pdf

$$f(x) = \frac{x^3 + 5x}{40}, \quad 1 \leq x \leq 3.$$

1. Verify that $f(x)$ is a pdf.
2. Find $P(X \geq 2)$.
3. Find $E(X)$.
4. Find $SD(X)$.
5. Find $F(x)$, the cumulative distribution function.
6. Find $\eta(0.8)$, or the 80th percentile.

Review Problem 2.2 A sample of 80 fibers is tested from this shipment; the sample breaking strength is found to be 49.1 N with a standard deviation of 5.2 N.

1. Find a 99% confidence interval for the mean breaking strength of the fibers.
2. Find a 99% confidence interval that gives a lower bound on the mean breaking strength of the fibers.
3. State an accurate interpretation of what the confidence interval actually means. Also, state at least one misinterpretation.

Review Problem 2.3 Brain weights are normally distributed with mean 1400.48 grams and standard deviation 106.33 grams. Find the 80th percentile of this distribution.

Review Problem 2.4 Vats labeled 30 L are filled with solution. The amount of solution put into each vat is random with mean 30.01 L and standard deviation 0.1 L. What is the probability that the total amount of solution contained in the 50 vats is more than 1500 L?

Review Problem 2.5 A die is rolled 4000 times. Estimate to four decimal places the probability of getting at least 700 sixes.

Review Problem 2.6 For a population with mean $\mu = 50$ and standard deviation $\sigma = 30$, what should the sample size n be in order to give you a standard error $\sigma_{\bar{X}}$ less than 3?

Review Problem 2.7 Describe how a probability plot is constructed, and how it can be used to determine if a sample follows the normal distribution.

Review Problem 2.8 A sample of 16 replicates of sludge from a wastewater plant has a mean pH of 6.68 with a standard deviation of 0.20. The probability plot indicates that the data approximately follows the normal distribution. Find a 95% confidence interval that gives an upper bound for the mean pH of the sludge.

Review Problem 2.9 Past experience has shown that 60% of all students in an introductory computer programming class have errors on their first assignment. In a class of 50 students, use the normal distribution to find the probability that between 25 and 33 students (inclusive) have errors on their first assignment.

Review Problem 2.10 Suppose a gambler plays roulette, betting on his favorite four numbers. If his number comes up (with probability $4/38$), then he wins \$8. However, if his number doesn't come up, he loses \$1.

1. Let X be the winnings on one play. Find $E(X)$ and $SD(X)$.
2. Suppose he plays 2500 times; let Y denote his total winnings. Find $E(Y)$ and $SD(Y)$.
3. If he plays 2500 times, find the probability that he has a positive net gain.

Review Problem 2.11 A chemist made 18 independent measurements of the melting point of tungsten. She obtained a sample mean of 3410.14°C and a sample standard deviation of 1.018°C .

1. Find a 99.9% two-sided confidence interval for the melting point of tungsten.
2. Find a 99.9% confidence interval that gives a lower bound for the melting point of tungsten.
3. Under the assumption of normality, find a 99.9% prediction interval for the next measurement.

Review Problem 2.12 A certain type of battery has a population mean lifetime of 40 hours with standard deviation 5 hours. Find the probability that the average lifetime of 100 such batteries is less than 38 hours.

Review Problem 2.13 A small college has 5,000 students, which consists of 47% men and 53% women. A simple random sample of 200 students is selected. What is the chance that fewer than 100 are men?

Review Problem 2.14 A soft-drink manufacturer purchases cans from an outside vendor. Of a random sample of 70 cans, 52 meet the specification for puncture resistance. Find a 95% confidence interval for the proportion of cans that meet the specification.

Review Problem 2.15 In the previous problem, find the sample size required to conservatively construct a 99% confidence interval to specify the proportion within ± 0.05 .

Review Problem 2.16 Let X be a normally distributed random variable with mean 100 and standard deviation σ . Suppose that you are told that $P(95 \leq X \leq 105) = 0.8124$. Find the value of σ .

Review Problem 2.17 Describe the essential difference between probability and statistics.