Math 1650 Homework Solutions
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§ 2.6

2 – 30 (even, except 12)

1 – 18 In these exercises you are asked to find a function that models a real-life situation. Use the guidelines for modeling described in the text to help you.

2) **Area** A poster is 10 inches longer than it is wide. Find a function that models its area A in terms of its width w.

The area of the poster is given by

$$\text{area} = \text{width} \times \text{height}.$$

Let w be the width of the poster, and we express height in terms of w.

In Words	In Algebra
Width	w
Height	$w + 10$

Thus we have

$$\begin{aligned} \text{area} &= \text{width} \times \text{height} \\ A(w) &= w(w+10) \\ A(w) &= w^2 + 10w \end{aligned}$$

4) **Volume** The height of a cylinder is four times its radius. Find a function that models the volume V of the cylinder in terms of its radius r.

The volume of a cylinder is given by

$$\text{volume} = \pi \times \text{radius}^2 \times \text{height}.$$

Let r be the radius of the cylinder, and we express height in terms of r.

In Words	In Algebra
Radius	r
Height	$4r$

Thus we have

$$\begin{aligned} \text{volume} &= \pi \times \text{radius}^2 \times \text{height} \\ V(r) &= \pi r^2 (4r) \\ V(r) &= 4\pi r^3 \end{aligned}$$

6) **Perimeter** A rectangle has an area of 16 m^2. Find a function that models its perimeter P in terms of the length x of one of its sides.

The perimeter of a rectangle is given by

Perimeter = 2(length + width).

Let x be the length of the rectangle, and we express width in terms of x.

In Words	In Algebra
Length	x
Width	$16/x$

Thus we have

$$\begin{aligned} \text{perimeter} &= 2(\text{length} + \text{width}) \\ P(x) &= 2\left(x + \frac{16}{x}\right) \\ V(r) &= 2\left(\frac{x^2 + 16}{x}\right) \end{aligned}$$

8) **Area** Find a function that models the surface area S of a cube in terms of its volume V.

The surface area of a cube is given by

surface area = 6(length of one side)^2.

Let l be the length of one side of the cube. We therefore need to express V, the volume of the cube, in terms of l. The formula that relates V and l is

$$V = l^3.$$

Solving for l, we get

$$l = \sqrt[3]{V}.$$

Thus we get

$$\begin{aligned} \text{surface area} &= 6(\text{length of one side})^2 \\ S(l) &= 6l^2 \\ S(V) &= 6\left(\sqrt[3]{V}\right)^2 \end{aligned}$$

10) **Area** Find a function that models the area A of a circle in terms of its circumference C.

The area of a circle is given by

$$\text{area} = \pi \times \text{radius}^2.$$

Let r be the radius of the circle. We therefore need to express C, the circumference of the circle, to r. The formula that relates C and r is

$$C = 2\pi r.$$

Solving for r we get

$$r = \frac{C}{2\pi}.$$

Thus we get

$$\begin{aligned} \text{area} &= \pi \times \text{radius}^2 \\ A(r) &= \pi r^2 \\ A(C) &= \pi\left(\frac{C}{2\pi}\right) = \frac{C^2}{4\pi} \end{aligned}$$

14) **Product** The sum of two positive numbers is 60. Find a function that models their product P in terms of x, one of the two numbers.

The product of two numbers is given by

$$\text{product} = (\text{first number}) \times (\text{second number}).$$

Let x be the first number. We therefore need to express y, the second number, in terms of x. The formula that relates x and y is

$$x + y = 60.$$

Solving for y gives us

$$y = 60 - x.$$

Thus we get

$$\begin{aligned} \text{product} &= (\text{first number}) \times (\text{second number}) \\ P(x) &= x(60 - x) \\ P(x) &= 60x - x^2 \end{aligned}$$

16) **Perimeter** A right triangle has one leg twice as long as the other. Find a function that models its perimeter P in terms of the length x of the shorter side.

The perimeter of a right triangle is given by

$$\text{perimeter} = (\text{short leg}) + (\text{long leg}) + (\text{hypotenuse}).$$

Let x be the length of the shorter side. We need to express the lengths of the longer leg and the hypotenuse in terms of x.

In Words	In Algebra
Short Leg	x
Long Leg	$2x$
Hypotenuse	$\sqrt{5}x$

Thus we get

$$\begin{aligned} \text{perimeter} &= (\text{short leg}) + (\text{long leg}) + (\text{hypotenuse}) \\ P(x) &= x + 2x + \sqrt{5}x \\ P(x) &= \left(3 + \sqrt{5}\right)x \end{aligned}$$

18) **Height** The volume of a cone in 100 in^3. Find a function that models the height h of the cone in terms of its radius r.

The volume of a cone is given by

$$\text{volume} = \frac{1}{3}\pi \times \text{radius}^2 \times \text{height}.$$

Therefore, by solving for height, we get

$$\text{height}=\frac{3 \times \text{volume}}{\pi \times \text{radius}^2}.$$

Let r be the radius of the cone, and since we already know $V = 100$, we get

$$h(r) = \frac{300}{\pi r^2}.$$

20) **Minimizing a Sum** Find two positive numbers whose sum is 100 and the sum of whose squares is a minimum.

Let x and y be the two numbers. The sum of their squares is given by

$$\text{sum of squares} = x^2 + y^2.$$

We will express y in terms of x. The formula that relates these two quantities is

$$x + y = 100.$$

Solving for y gives us

$$y = 100 - x.$$

Thus we have

$$\begin{aligned} \text{sum of squares} &= x^2 + y^2 \\ S(x) &= x^2 + (100 - x)^2 \\ S(x) &= 2x^2 - 200x + 10000 \end{aligned}$$

Since $a = 2, b = -200, c = 10000$, and $a > 0$, we see that this quadratic function has a minimum value at

$$x = -\frac{b}{2a} = -\frac{-200}{4} = 50.$$

Thus $x = 50$ and $y = 100 - x = 100 - 50 = 50$.

22) **Maximizing Area** Among all rectangles that have a perimeter of 20 ft, find the dimensions of the one with largest area.

Let w and l be the width and length of the rectangle respectively. Thus we have

$$\text{area} = l \times w.$$

We will express l in terms of w. The formula that relates these two quantities is

$$2(l + w) = 20.$$

Solving for l we get

$$l = 10 - w.$$

Thus we have

$$\begin{aligned} \text{area} &= l \times w \\ A(w) &= (10 - w)w \\ A(w) &= 10w - w^2 \end{aligned}$$

Since $a = -1, b = 10$, and $a < 0$, we see that this quadratic function has a maximum value at

$$w = -\frac{b}{2a} = -\frac{10}{-2} = 5.$$

Thus $w = 5$ and $l = 10 - w = 10 - 5 = 5$.

24) **Dividing a Pen** A rancher with 750 ft of fencing wants to enclose a rectangular area and then divide it into four pens with fencing parallel to one side of the rectangle.

(a) Find a function that models the total area of the four pens.

(b) Find the largest possible total area of the four pens.

Let w and l be the width and length of the rectangular area. Thus we have

$$\text{area} = w \times l.$$

We will express l in terms of w. The formula that relates l and w together is

$$5w + 2l = 750.$$

Solving for l, we get

$$l = \frac{750 - 5w}{2}.$$

Thus, we have

$$\begin{aligned} \text{area} &= l \times w \\ A(w) &= \left(\frac{750 - 5w}{2}\right) w \\ A(w) &= 375w - \frac{5}{2}w^2 \end{aligned}$$

Since $a = -\frac{5}{2}, b = 375, c = 0$, and $a < 0$, we see that this quadratic function has a maximum value of

$$c - \frac{b^2}{4a} = 0 - \frac{375^2}{4\left(-\frac{5}{2}\right)} = \frac{140625}{10} = 14{,}062.5\ ft^2.$$

26) **Maximizing Area** A wire 10 cm long is cut into two pieces, one of length x and the other of length $10-x$, Each piece is bent into the shape of a square.

(a) Find a function that models the total area enclosed by the two squares.

(b) Find the value of x that minimizes the total area of the two squares.

The function that models the total area of the two squares is given by

$$\begin{aligned} A(x) &= x^2+(10-x)^2 \\ &= x^2+(100-20x+x^2) \\ &= 2x^2-20x+100. \end{aligned}$$

Since $a=2, b=-20$, and $a>0$, we see that this quadratic function has a minimum value at

$$x=-\frac{b}{2a}=-\frac{-20}{2(2)}=\frac{20}{4}=5.$$

28) **Maximizing Profit** A community bird-watching society makes and sells simple bird feeders to raise money for its conservation activities. The materials for each feeder cost \$6, and they sell an average of 20 per week at a price of \$10 each. They have been considering raising the price, so they conduct a survey and find that for every dollar increase they lose 2 sales per week.

(a) Find a function that models weekly profit in terms of price per feeder.

(b) What price should the society charge for each feeder to maximize profits? What is the maximum profit?

Let x be the price per feeder and note that the profit per feeder is given by the formula

$$x-6.$$

So that total profit would be modeled by

$$P(x)=(x-6)N(x),$$

where $N(x)$ is the number of feeders sold at price x. Thus we need to find a formula for $N(x)$.

The formula for $N(x)$ is given as

$$\begin{aligned} N(x) &= 20 - 2(x-10) \\ &= 20 - 2x + 20 \\ & \quad 40 - 2x. \end{aligned}$$

Thus

$$\begin{aligned} P(x) &= (x-6)(40-2x) \\ &= 40x - 2x^2 - 240 + 12x \\ &= -2x^2 + 52x - 240. \end{aligned}$$

Since $a = -2, b = 52, c = -240$, and $a < 0$, we see that this quadratic function has a maximum value of

$$c - \frac{b^2}{4a} = -240 - \frac{52^2}{4(-2)} = -240 + \frac{2704}{8} = -240 + 338 = 98,$$

this maximum profit is attained at

$$x = -\frac{b}{2a} = -\frac{52}{2(-2)} = 13.$$

30) **Volume of a Box** A box with an open top is to be constructed from a rectangular piece of cardboard with dimensions 12 in. by 20 in. by cutting out equal squares of side x at each corner and then folding up the sides.

(a) Find a function that models the volume of the box.

(b) Find the values of x for which the volume is greater than 200 in^3.

(c) Find the largest volume that such a box can have.

The volume of a box is given by

$$\text{volume} = \text{length} \times \text{width} \times \text{height}$$

We need to express length. width, and height in terms of x. Doing so, we get

In Words	In Algebra
Width	$12 - 2x$
Length	$20 - 2x$
Depth	x

Thus we have

$$\begin{aligned} V(x) &= (12 - 2x)(20 - 2x)x \\ &= 240x - 24x^2 - 40x^2 + 4x^3 \\ &= 4x^3 - 64x^2 + 240x. \end{aligned}$$

Using a graphing calculator, we see that for $1.17379 < x < 3.89786$, the volume of the box is greater than 200 in^3. Also using a calculator, we find that the largest volume of such a box is 262.682 in^3.