§ 2.2: Graphs of Functions

Graphing Functions

The Graph of a Function

If $f$ is a function with domain $A$, then the graph of $f$ is the set of ordered pairs

$$\{(x, f(x)): x \in A\}$$

In other words, the graph of $f$ is the set of all points $(x, y)$ such that $y = f(x)$; that is, the graph of $f$ is the graph of the equation $y = f(x)$.

Example 1

Sketch the graph of the following functions.

(a) $f(x) = x^2$

(b) $g(x) = x^3$

(c) $h(x) = \sqrt{x}$

(d) $i(x) = |x|$
Getting Information from the Graph of a Function

The graph of a function helps us picture the domain and range of the function on the x-axis and y-axis as shown below.

Example 2  Finding the Domain and Range from a Graph

Below is the graph of the function \( f(x) = \sqrt{4 - x^2} \). Find the domain and range of \( f \).
Graphing Piecewise Defined Functions

Example 3

Graph of a Piecewise Defined Function

Sketch the graph of the function

\[
f(x) = \begin{cases} 
  x^2 & x \leq 1 \\
  2x + 1 & x > 1 
\end{cases}
\]

The greatest integer function is defined by

\[ [x] = \text{greatest integer which is less than } x \]

Example 4

Graph of the Greatest Integer Function

Sketch the graph of \( f(x) = [x] \).
The Vertical Line Test

A curve in the coordinate plane is the graph of a function if and only if no vertical line intersects the curve more than once.

Example 5  Using the Vertical Line Test

Using the vertical line test, determine which of the following curves are graphs of functions.
Example 6  Equations That Define Functions

Do the following equations define $y$ as a function of $x$?

(a) $y - x^2 = 2$

(b) $x^2 + y^2 = 4$

Homework

Due:________________________________________

2–24 (even), 38–50 (even), 56, 62–72 (even)