

Math 1710 Section 3
Midterm 2 - Review

I.

1. A man 6 ft tall walks at the rate of 5 ft/sec toward a streetlight that is 16 ft above the ground. At what rate is the tip of his shadow moving? At what rate is the length of his shadow changing when he is 10 ft from the base of the light?
2. A spherical balloon is inflated with helium at the rate of 100π ft³/min. How fast is the balloon's radius increasing at the instant the radius is 5 ft? How fast is the surface area increasing?

II. Find the heights of the absolute maxima and minima of the following functions over the given intervals.

3. $f(x) = \tan x$, $[\frac{-\pi}{3}, \frac{\pi}{4}]$
4. $g(x) = 2 - |x|$, $[-1, 3]$
5. $h(x) = x^3 + x^2 - 8x + 5$, $[-4, 2]$

III.

6. Suppose that a ball is thrown vertically into the air from a height of 6 ft and with an initial velocity of 40 ft/sec. Given that height of the ball in feet t seconds after release is $h(t) = -16t^2 + 40t + 6$, how high will the ball be at its apex and how many seconds will it take to get there?
7. Suppose that after t seconds the current i (measured in amperes) in an alternating current circuit is $i = 2 \cos t + 2 \sin t$. What is the largest amount of current that will ever be in the circuit?

IV. Using the first and second derivative tests, graph the following functions. Label all local extrema and inflection points.

8. $f(x) = x^4 + 2x^3$
9. $g(x) = x + \sin x$ (only graph over $[0, 2\pi]$ and label the endpoints)
10. $h(x) = \frac{x^2-3}{x-2}$
11. $f(x) = 5x^{\frac{2}{5}} - 2x$

V.

12. Consider a box with a square end such that the sum of its length and its girth is 108 in. What dimensions will give the box the largest possible volume?
13. Consider a hollow cylindrical container that holds 1000 cm³. What dimensions will give it the smallest surface area?

VI. Use L'Hopital's Rule to find the following limits.

14. $\lim_{x \rightarrow 0} \frac{x(\cos x - 1)}{\sin x - x}$
15. $\lim_{x \rightarrow 1} \frac{5x^2 - 5}{x - 1}$

16. $\lim_{x \rightarrow 0^+} \left(\frac{1}{x} - \frac{1}{\sqrt{x}} \right)$

17. $\lim_{x \rightarrow -\infty} \frac{3x-5}{2x^2-x+2}$

18. $\lim_{x \rightarrow 0} \frac{\sin 7x}{\tan 11x}$

VII.

19. Starting with $x_0 = 3$, estimate a solution of $\tan x = 0$ by calculating x_3 .

20. Starting with $x_0 = -\pi$, estimate a solution of $\cos x = 2x$ by calculating x_2 .

VIII. Find functions that satisfy the given information.

21. $f'(x) = \cos(\pi x)$, $f(0) = 1$

22. $g''(x) = 2 - 6x$, $g'(0) = 4$, $g(0) = 1$

23. $h''(x) = 0$, $h'(0) = 2$, $h(0) = 0$

24. $f''(x) = \frac{2}{x^3}$, $f'(1) = 1$, $f(1) = 1$

25. $g^{(4)}(x) = -\sin x + \cos x$, $g'''(0) = 7$, $g''(0) = g'(0) = -1$, $g(0) = 0$