This exam will be formatted like the first one:

- Notes, books, and calculators will not be allowed.
- For full credit, show enough work to explain your method.
- The problems will mainly be computational, and none will be proof-based.

Here is a list of topics, with the relevant sections. For preparation, redo some of the problems from Homeworks 6, 7, 8, and 9, and if you have time, do some of the additional practice problems given here.

Pseudoinverses

These will not be tested on this exam, as we will cover them more completely when we study the singular value decomposition.

Determinants

- Know the cofactor formula for the determinant.
- Know the behaviour of the determinant under row and column operations.
- Know that the determinant distributes over matrix multiplication.
- Know that the determinant is the product of the pivots.
- Section 5.3, on determinants as volumes, will not be tested.
- *Problems:* Section 5.1: 2, 3, 5, 8; Section 5.2: 2, 4, 10, 12, 13.

Eigenvectors and eigenvalues

- Be able to compute the characteristic polynomial and the eigenvalues.
- The trace and determinant are the sum and product of the eigenvalues.
- Be able to compute eigenvectors and bases of eigenspaces.
- *Problems:* Section 6.1: 9, 11, 18, 21, 23, 25, 27, 29, 30, 31.

Diagonalization

- A is diagonalizable if and only if for every eigenvalue λ, the dimension of $\mathrm{Null}(A-\lambda)$ is the multiplicity of λ as a root of $\mathrm{char}_A(t)$.
- For A diagonalizable, be able to find X and Λ such that $A = X\Lambda X^{-1}$.
- Be able to use $A = X\Lambda X^{-1}$ to compute A^k.
- If $A = YBY^{-1}$, then A and B have the same eigenvalues.
- *Problems:* Section 6.2: 4, 5, 6, 11, 12, 16, 17, 25, 28.

Symmetric matrices

Given a symmetric matrix S:

- Be able to diagonalize it by an orthogonal matrix: $S = Q\Lambda Q^T$.
- Know how to compute the projection form of the spectral theorem:

  $$S = \lambda_1 P_{V_1} + \lambda_2 P_{V_2} + \cdots + \lambda_r P_{V_r},$$

  where $\lambda_1, \ldots, \lambda_r$ are the *distinct* eigenvalues, and V_s is the λ_s-eigenspace.
- *Problems:* Section 6.3: 14, 15, 16, 17, 18.

Positive definite and positive semidefinite matrices

Let S be a symmetric matrix.

- Definition: S is PDS (PSDS) if all eigenvalues are positive (nonnegative).
- Test: S is PDS (PSDS) iff $x^T Sx$ is positive (nonnegative) for all $x \neq 0$.
- Test: S is PDS iff all upper left subdeterminants are positive.
- If S is PDS, then all diagonal subdeterminants are positive.
- Test: S is PDS iff it has an LU-decomposition with positive pivots.
- $A^T A$ is always PSDS, and it is PDS iff A's columns are independent.
- For S PDS, be able to compute its Cholesky decomposition $C^T C$.
- For S PSDS, be able to compute its PSDS square root.
- For S 2×2 PDS, be able to graph $x^T Sx = \gamma$ for any $\gamma > 0$.
- *Problems:* Section 6.3: 4, 24, 26, 29, 30, 34, 35, 37, 40, 41, 42.

Orthogonal motions of $\mathbb{R}^2$ and $\mathbb{R}^3$

- Know Propositions 6.3 and 6.4 in the supplemental notes.
- For S 2×2 symmetric, be able to find all four orthogonal matrices diagonalizing it, and to describe geometrically the way they act on $\mathbb{R}^2$.