Math 1680 Class Notes

Chapter 16. The Law of Averages

- Misconception: with a good sample, the sample percentage will equal the population percentage.

- number of heads = half the number of tosses + chance error

- Coin tossing: if we toss a coin 10,000 times, are we likely to get exactly 5,000 heads? Surely not. As the number of tosses goes up, the difference between the number of heads and the expected number tends to get larger and larger in absolute terms, that is , as a number. However, the difference tends to get smaller and smaller in percentage terms, relative to the number of tosses. (R4)

- *Box models and sum of the draws from a box.* A box model consists of draws made at random from a box of tickets; each ticket in the box shows a number. The chance variability in coins, dice, roulette wheels (and later, sampling processes) is related to the chance variability in draws from a box. This produces real economy of thought: there is a general theory, instead of a lot of special cases. (B6,R1: distinction between absolute and relative errors)

- Gambling at roulette: the sum of the draws from the box corresponds to the net gain. (Example 1, C2,R8,R10)

Chapter 17. The Expected Value and Standard Error

- The sum of draws made at random with replacement from a box will be around its expected value, but will be off by a chance error:

$$\text{observed sum} = \text{expected value} + \text{chance error}.$$

- The expected value for the sum equals (number of draws) × (average of box).

- The likely size of the chance error is given by the SE for the sum. Chance errors of an SE or so in size are fairly common, but chance errors bigger than several SEs in size are very unusual. The SE for the sum is computed by the square root law as

$$\sqrt{\text{number of draws}} \times \text{SD of box}.$$

(R12)

- What the square root means: when the number of draws goes up by a factor of 100, say, the SE for the sum of the draws only goes up by the factor $\sqrt{100} = 10$. In particular, as the number of draws goes up, the SE for the sum goes up in absolute terms, but goes down relative to the number of draws. This is the mathematical explanation for the law of averages. (C8 reinforces the law of averages)

- When the number of draws is large, the normal approximation can be used. (R4,9,10)

  (note on R9: two games with the same expected value need not offer the same chance of winning; this exercise should demonstrate why the SE is needed.) (note on R10: False; square root law)

- Expected values are not random variables that are computed up to some margin of error. (B6)

- Many boxes in gambling problems (roulette, for instance) have only two kinds of tickets, and there is a short cut formula for the SD of the box:

$$\left(\begin{array}{c}\text{big}\\ \text{number}\end{array} - \begin{array}{c}\text{small}\\ \text{number}\end{array}\right) \times \sqrt{\begin{array}{c}\text{fraction with}\\ \text{big number}\end{array} \times \begin{array}{c}\text{fraction with}\\ \text{small number}\end{array}}$$

- Classifying and counting: if you have to classify and count the draws, put 0's and 1's on the tickets. Mark 1 on the tickets that count for you, 0 on the others. (E7: change the 1's to red, 0's to green) (R11)

Chapter 18. The Normal Approximation for Probability Histograms

- Probability calculations for sums through the normal curve: when the number of draws is large, there is about a 68% chance for the sum to be within one SE of the expected value, and so on.

- Probability histogram: a graph which represents chance by area. It is the limit of empirical histograms from simulations. (figure 1 and 2) Figure 2 shows that not everything is normally distributed. The normal curve is tied to sums. (R3)

- Don't confuse the probability histogram for the sum with the histogram for the data—the tickets from the box.

- Central Limit Theorem: the probability histogram for the sum of a large number of draws from a box will follow the normal curve very closely. However, if the distribution of tickets in the box is highly skewed, then many draws may be needed before the approximation takes hold. (R9,10: you get a feeling for how many draws are enough by looking at the pictures in the chapter.)

- Continuity correction: to estimate the chance that the sum will take a given value. (R4,6)

- *Note on the SD.* The normal approximation shows why the SD is so useful. The shape of the probability histogram for the sum of a large number of draws from a box depends only on the average and SD of the numbers in the box. Other measures of spread, like average absolute deviation from average, have very little to do with it. (See note 9 to the chapter) (R11)