Math 1100 — Test #2
Summer II 2002

Instructions. To get full credit, you must show your work. Good luck!

1. (16 pts.) The graph of a function $f$ is shown in the following figure.

(a) State the domain and range of $f$.
(b) State the values of $f(-1)$, $f(0)$, and $f(4)$.
(c) For which values of $x$ is $f(x) = 0$?
(d) Which is larger, $f(1)$ or $f(2)$?

2. (12 pts.) $P(x) = x^4 - 2x^3 + x^2 - x - 2$

(a) Use synthetic division and the Remainder Theorem to evaluate $P(-2)$.
(b) Use Factor Theorem to determine whether $x + 1$ is a factor of $P(x)$.
(c) Is 2 a zero of $P(x)$? Explain.

3. (12 pts.) Sketch the graph of the polynomial function $P(x) = x^2(x + 1)(x - 1)^2$.
Make sure your graph shows all intercepts and exhibits the proper end behavior.

4. (12 pts.) $f(x) = -4x^2 - 16x + 3$

(a) Express the quadratic function in standard form.
(b) Sketch the graph of $f$.
(c) Find the maximum or minimum value of $f(x)$.
5. (16 pts.) \( f(x) = 2 - x^2 \)

(a) Sketch the graph of \( f \).
(b) Find the domain and the range of \( f \).
(c) Determine the average rate of change of \( f \) between \( x = 2 \) and \( x = 3 \).
(d) Find the intervals on which \( f \) is increasing and on which \( f \) is decreasing.

6. (10 pts.) If \( f(x) = x^2 - 6 \) and \( g(x) = \frac{4}{x-2} \), find \( (f + g)(4) \) and \( (f - g)(4) \).

7. (12 pts.) Find all rational zeros of the polynomial \( P(x) = 2x^3 + x^2 - 5x + 2 \) and sketch the graph of \( P \).

8. (10 pts.) \( s \) is inversely proportional to the square root of \( t \). If \( s = 100 \), then \( t = 25 \).

(a) Express the statement as a formula.
(b) Use the given information to find the constant of proportionality.

9. (Bonus: 10 pts.) The cost of a sheet of gold foil is proportional to its area. If a rectangular sheet measuring 15 cm by 20 cm costs $75, how much would a 3 cm by 5 cm sheet cost?

10. (Bonus: 10 pts.) Given \( f(x) = \sqrt{x-5} - 5 \) and \( g(x) = \frac{1}{x-6} \), find \( fg \), \( \frac{f}{g} \), and \( f \circ g \) and their domains.