Math 1100 Test #2 (Answers)

1. (16 pts.)
   (a) The domain is \([-4, 6]\) and the range is \([-3, 3]\).
   (b) \(f(-1) = 0, f(0) = -1,\) and \(f(4) = -1\)
   (c) \(x = -1\) and \(x = 5\)
   (d) \(f(1) = -2\) and \(f(2) = -3;\) so \(f(1)\) is larger.

2. (12 pts.) \(P(x) = x^4 - 2x^3 + x^2 - x - 2\)
   (a) \[
   \begin{array}{c|ccccc}
   x & 1 & -2 & 1 & -1 & -2 \\
   \hline
   2 & 1 & -2 & 1 & -1 & -2 \\
   \end{array}
   \]
   By synthetic division, the remainder of \(P(x) / (x + 2)\) is 36. So by the Remainder Theorem, \(P(-2) = 36\).
   (b) Since \(P(-1) = 1 + 2 + 1 + 1 - 2 = 3,\) by Factor Theorem, \(x + 1\) is not a factor of \(P(x)\).
   (c) \[
   \begin{array}{c|ccccc}
   x & 1 & -2 & 1 & -1 & -2 \\
   \hline
   2 & 1 & -2 & 1 & -1 & -2 \\
   \end{array}
   \]
   Since the remainder of \(P(x) / (x - 2)\) is zero, we have \(P(2) = 0.\) Therefore 2 is a zero of \(P(x)\).

3. (12 pts.) The zeros of \(P(x)\) are \((-1, 0, 1).\) The \(y\)-intercept is \((0, 0).\) For additional points of the graph, we have \(f(-2) = -36, f(-\frac{1}{2}) = \frac{9}{32}, f(\frac{1}{2}) = \frac{3}{32},\) and \(f(2) = 12.\)

4. (12 pts.) \(f(x) = -4x^2 - 16x + 3\)
   (a) \(f(x) = -4(x^2 + 4x + 4) + 3 + 16 = -4(x + 2)^2 + 19\)
   (b) \[
   \begin{array}{c|ccccc}
   x & -4 & -2 & 0 & 2 & 4 \\
   \hline
   \end{array}
   \]
   (c) The maximum value of \(f(x)\) is 19.
5. (16 pts.) \( f(x) = 2 - x^2 \)

(a) 

(b) The domain is \((-\infty, \infty)\) and the range is \((-\infty, 2]\).

(c) \( \frac{f(0) - f(2)}{3-2} = -7 - (-2) = -5 \)

(d) \( f \) is increasing on \((-\infty, 0)\) and \( f \) is decreasing on \((0, \infty)\).

6. (10 pts.) \((f + g)(4) = 4^2 - 6 + \frac{4}{3-2} = 10 + 2 = 12\) and \((f - g)(4) = 10 - 2 = 8\).

7. (12 pts.) The possible rational zeros are \(\pm 1, \pm 2, \pm \frac{1}{2}\). By synthetic division, we have

\[
\begin{array}{c|ccccc}
 & 1 & 2 & 1 & -5 & 2 \\
\hline
1 & 1 & 3 & -2 & 0 \\
\end{array}
\]

Therefore \( P(x) = (x - 1)(2x^2 + 3x - 2) = (x - 1)(2x - 1)(x + 2) \). So the rational zeros are \(\{-2, \frac{1}{2}, 1\}\). The graph of \( P \) is

8. (10 pts.) \( s \) is inversely proportional to the square root of \( t \). If \( s = 100 \), then \( t = 25 \).

(a) \( s = \frac{k}{\sqrt{t}} \)

(b) \( 100 = \frac{k}{\sqrt{25}} \Rightarrow k = 500 \)

9. (Bonus: 10 pts.) Let \( C \) be the cost of the sheet of gold foil and \( A \) be its area. Then \( C = kA \) for some non-zero constant \( k \). Using the given information, we have \( 75 = k(15)(20) \Rightarrow k = 0.25 \). So the formula for \( C \) is \( C = 0.25A \). Therefore a 3 cm by 5 cm sheet would cost \( 0.25(3)(5) = 3.75 \) dollars.

10. (Bonus: 10 pts.) \((fg)(x) = \sqrt{\frac{31 - 5x}{x-6}} \) with domain \([5, 6) \cup (6, \infty)\); \((f/g)(x) = \left(\sqrt{x - 5} - 5\right)(x - 6) \) with domain \([5, 6) \cup (6, \infty)\); \((f \circ g)(x) = f\left(\frac{1}{5-x}\right) = \sqrt{\frac{1}{5-x} - 5} - 5 \). The domain of \( f \circ g \) is the set of all \( x \) in \((-\infty, 6) \cup (6, \infty) \) such that \( \frac{1}{5-x} \geq 5 \). Note that \( \frac{1}{5-x} \geq 5 \Leftrightarrow \frac{1}{5-x} - 5 \geq 0 \Leftrightarrow \frac{1-5x+30}{5-x} \geq 0 \Leftrightarrow \frac{31-5x}{5-x} \geq 0 \)

<table>
<thead>
<tr>
<th>sign of ( 31 - 5x )</th>
<th>((-\infty, 6))</th>
<th>((6, \frac{31}{5}))</th>
<th>((\frac{31}{5}, \infty))</th>
</tr>
</thead>
<tbody>
<tr>
<td>sign of ( x - 6 )</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>sign of ( \frac{31-5x}{x-6} )</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
</tbody>
</table>

From the signs chart, the solution of \( \frac{31-5x}{x-6} \geq 0 \) is \((6, \frac{31}{5})\).

Therefore the domain of \( f \circ g \) is \((6, \frac{31}{5})\).