Divisibility Tests

Most people know an easy way to check if a number is divisible by 2, 5 or 10. The purpose of this project is for you to discover how to check if a number is divisible by other small numbers. Follow the steps below to complete the project.

1. The first step is to verify the usual test for divisibility by 2.
   a. Expand the number 5719 into its base 10 expansion.
   b. To the expansion in part a, apply the rules to evaluate a number modulo 2. Based on your observation, why is only the ones place important in determining if a number is divisible by 2?
   c. Suppose that a number is written base 10 as $a_0 + a_1 \times 10 + a_2 \times 10^2 + \cdots + a_n \times 10^n$. Show how using arithmetic modulo 2 gives the usual rule telling if a number is divisible by 2.
2. Repeat number 1, but this time use 5 instead of 2.
3. Repeat number 1, but this time use 10 instead of 5.
4. Now find a test for divisibility by 3 by following the steps below.
   a. Expand the number 29412 into its base 10 representation. Rewrite what the number is mod 3 by replacing 10 and all its powers with their mod 3 equivalents, but leaving the digits in the number as they are. Next simplify by reducing further mod 3 until you determine a number between 0 and 2 inclusive which is equivalent to 2941 mod 3.
   b. Repeat part a, using the number 49102 instead of 29412.
   c. Based on your calculations in parts b and c, state a rule about how to check divisiblity by 3.
   d. Show that your rule always works in the same way you verified the divisibility test for 2 in number 1 part c.
5. Repeat the process in number 4, using 9 instead of 3.
6. Repeat the process in number 4, using 11 instead of 3.
7. Using the same ideas as above state and show the divisibility tests for 4 and 8.
8. Summarize the divisibility tests for 2,3,4,5,6,8,9,10, and 11. Include examples. (In the case of 6, you can just check divisible by 3 and divisible by 2. Why?)
9. In part 8 you listed simple ways to check divisibility by all the number from 2 to 11 inclusive except for the number 7. Use the same procedure as you used above to find a way to check that a number is divisible by 7. The divisibility test for 7 you develop will probably be a little more complicated than the other tests.