In this assignment you will find a relatively simple formula for the n^{th} Fibonacci number. The Fibonacci numbers are defined by the difference equation $x_n = x_{n-1} + x_{n-2}$ and the initial conditions $x_0 = 0$ and $x_1 = 1$. The Fibonacci sequence starts $0, 1, 1, 2, 3, 5, 8, 13, 21, 24, \ldots$ I think you will agree that a formula for the n^{th} Fibonacci number is not obvious. After you derive it, you will see why.

Use the techniques developed in class to solve the finite difference equation with the given initial conditions. You will find that the numbers do not come out even, but if you are careful and persevere, you will be rewarded with a correct formula. Check some of the numbers using your calculator.

You can simplify your answer a bit. Your answer should be the sum of two terms. Which is the larger term? How small is the smaller term? Since the Fibonacci numbers are integers, do you really need to compute the small term in order to determine the Fibonacci numbers? Explain.

Find the successive ratios x_n/x_{n-1} for $n = 1, 2, 3, 4, 5, 6, 7, 8, 9$. Use the formula to find the ratio for $n = 100$. Use the concepts of eigenvalues and eigenvectors to explain why this makes sense.