1. Compute $\det \begin{bmatrix} 2 & 3 \\ 1 & 6 \end{bmatrix}$

2. Compute $\det \begin{bmatrix} 1 & -2 & 3 \\ 2 & 4 & 1 \\ -2 & 3 & -3 \end{bmatrix}$

3. Compute $\det \begin{bmatrix} 1 & 0 & 3 & 2 \\ -1 & 2 & 4 & 1 \\ 2 & 0 & 3 & 0 \\ 5 & 0 & 0 & 2 \end{bmatrix}$

4. Compute $\det \begin{bmatrix} 3 & 2 & 2 & 9 & 1 \\ 0 & 2 & 4 & 3 & -1 \\ 0 & 0 & 3 & 0 & -5 \\ 0 & 0 & 0 & 6 & 2 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$

5. Compute $\det \begin{bmatrix} 1 & 4 & 6 & 2 & 9 \\ 0 & 3 & -1 & 2 & -1 \\ 0 & 0 & 3 & 4 & 6 \\ 0 & 0 & 0 & -1 & 2 \\ 0 & 0 & 3 & 2 & 4 \end{bmatrix}$

6. You are given that $\det \mathbf{A} = 5$ and $\det \mathbf{B} = -2$. Find $\det \mathbf{A}^3\mathbf{B}^2$.

7. You are given that $\det \mathbf{A} = -1$ and $\det \mathbf{B} = 4$. Does $\mathbf{A}^{-1}$ exist? If so, compute $\det \mathbf{B}^2\mathbf{A}^{-2}\mathbf{B}$.

8. What happens to the determinant of a matrix if you change the sign of all the entries of the matrix? Explain.

9. What happens to a determinant if you switch two rows of the matrix? Switch two columns?

10. Write the formula for Cramer's Rule. Use it to solve the system

$$\begin{aligned} -5x &+ 3y &= 9 \\ 3x &- y &= -5. \end{aligned}$$

11. Dervie the formula for Cramer's Rule.

12. Define the terms eigenvalue and eigenvector for a square matrix $\mathbf{A}$.

13. Is $\begin{bmatrix} 2 \\ -7 \end{bmatrix}$ an eigenvector for the matrix $\begin{bmatrix} -8.5 & -4.5 \\ 21 & 11 \end{bmatrix}$? Explain.

14. Show that if $\mathbf{v}$ is an eigenvector for both $\mathbf{A}$ and $\mathbf{B}$, then $\mathbf{v}$ is an eigenvector for $\mathbf{AB}$. If the eigenvalues for $\mathbf{v}$ are $\lambda_{\mathbf{A}}$ and $\lambda_{\mathbf{B}}$ for the matrices $\mathbf{A}$ and $\mathbf{B}$ respectively, find the eignvalue corresponding to $\mathbf{v}$ with respect to the matrix $\mathbf{AB}$.

15. Find the eigenvalues and eigenspaces for the matrix $\begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$.

16. Find the eigenvalues and eigenspaces for the matrix $\begin{bmatrix} 10 & -4 & 15 \\ 8 & -2 & 15 \\ -4 & 2 & -5 \end{bmatrix}$.

17. Find all the eigenvalues of the matrix $\mathbf{A} = \begin{bmatrix} 1 & 5 \\ -2 & 3 \end{bmatrix}$.

18. If possible, diagonalize the matrix $\begin{bmatrix} 5 & 2 \\ -24 & -9 \end{bmatrix}$. If it is not possible, say why.

19. If possible, diagonalize the matrix $\begin{bmatrix} 12 & -16 & 0 \\ 8 & -12 & 0 \\ 6 & -12 & 2 \end{bmatrix}$. If it is not possible, say why.

20. If possible, diagonalize the matrix $\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$. If it is not possible, say why.

21. Are the matrices $\begin{bmatrix} 3 & 0 \\ 2 & 3 \end{bmatrix}$ and $\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$ similar? How do you know?

22. Are the matrices $\begin{bmatrix} 3 & 0 \\ 2 & 2 \end{bmatrix}$ and $\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$ similar? How do you know?

23. Solve the finite diffference equation $x_n = x_{n-1} + 2x_{n-2}$ with initial conditions $x_0 = 0$ and $x_1 = 1$.

24. Solve the finite difference equation $x_n = x_{n-1} + 12x_{n-2}$ with initial conditions $x_0 = 1$ and $x_1 = 3$.

25. A population consists of youth and adults. The Leslie matrix for the population is $\begin{bmatrix} 15 & 7.5 \\ 0.8 & 0 \end{bmatrix}$, where the first column represents youth and the second column represents adults. Use eignvalues to determine the long range growth rate of this population.

26. A matrix $\mathbf{A}$ has eigenvalues 2 and 3 with eignevectors $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$, respectively. Compute $\mathbf{A}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\mathbf{A}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$. Determine the matrix $\mathbf{A}$.

27. Show that if $\mathbf{A}$ is similar to $\mathbf{B}$, then $\mathbf{A}^2$ is similar to $\mathbf{B}^2$.

28. Show that if $\mathbf{A}$ is similar to $\mathbf{B}$ and $\mathbf{A}$ is invertible, then $\mathbf{B}$ is invertible and $\mathbf{A}^{-1}$ is similar to $\mathbf{B}^{-1}$.

29. Show that if $\mathbf{A}$ is similar to $\mathbf{B}$ and $\mathbf{B}$ is similar to $\mathbf{C}$, then $\mathbf{A}$ is similar to $\mathbf{C}$.