1. Compute the inverse of the matrix
$$\begin{bmatrix} 3 & 7 \\ 8 & -2 \end{bmatrix}$$
or else explain why the inverse does not exist.

2. Compute the inverse of the matrix
$$\begin{bmatrix} 0 & -1 & 0 \\ -2 & 2 & 1 \\ 1 & 2 & 0 \end{bmatrix}$$
or else explain why the inverse does not exist.

3. Solve the system
$$\begin{bmatrix} 0 & -1 & -3 \\ 1 & 5 & 16 \\ 1 & 2 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix}$$
by computing a matrix inverse.

4. Suppose that $\mathbf{AB} = \mathbf{I}$, where each matrix is $n \times n$. Show that for any vector $b \in \mathbf{R}^n$, the equation $\mathbf{A}x = b$ has a solution.

5. Suppose all the entries in the second column of $\mathbf{B}$ are 0. What (if anything) does this say about the entries of $\mathbf{AB}$? What about $\mathbf{BA}$?

6. Suppose that $\mathbf{A}, \mathbf{B}, \mathbf{C}$ are all $n \times n$ invertible matrices. Give the formula for $(\mathbf{ABC})^{-1}$ in terms of $\mathbf{A}^{-1}$, $\mathbf{B}^{-1}$ and $\mathbf{C}^{-1}$. Verify that your answer is correct.

7. Solve the equation $\mathbf{AB} = \mathbf{BC}$ for $\mathbf{A}$ assuming that $\mathbf{A}, \mathbf{B}$, and $\mathbf{C}$ are $n \times n$ matrices and $\mathbf{B}$ is invertible.

8. Suppose that $\mathbf{C}$ is invertible and that $\mathbf{A} = \mathbf{CBC}^{-1}$. Solve for $\mathbf{B}$.

9. Suppose that $\mathbf{C}$ and $\mathbf{B}$ are invertible and that $\mathbf{A} = \mathbf{CBC}^{-1}$. Find $\mathbf{A}^{-1}$ and verify that your answer is correct.

10. Give the definition of the inverse for a square matrix $\mathbf{A}$.

11. Let $\mathbf{A}$ be a square matrix. List eight conditions equivalent to $\mathbf{A}$ being invertible.

12. A linear transformation $T : \mathbf{R}^2 \to \mathbf{R}^2$ is given by the formula
$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 3x + 4y \\ 5x + 7y \end{bmatrix}.$$
Does the inverse T^{-1} exist? If so, compute the inverse. If not, say how you know.

13. A linear transformation $T : \mathbf{R}^3 \to \mathbf{R}^3$ is given by the formula
$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x - y - z \\ 5x - y - 7z \\ 4x - 2y - 6z \end{bmatrix}.$$
Does the inverse T^{-1} exist? If so, compute the inverse. If not, say how you know.

14. A linear transformation $T : \mathbf{R}^2 \to \mathbf{R}^3$ is given by the formula
$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x + y \\ 2x - y \\ -3x + 4y \end{bmatrix}.$$

Does T^{-1} exist? If so, compute the inverse. If not, say how you know.

15. A linear transformation $T : \mathbf{R}^3 \to \mathbf{R}^2$ is given by the formula
$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x - 2y - z \\ x + 4y - 7z \end{bmatrix}.$$
Does the inverse T^{-1} exist? If so, compute the inverse. If not, say how you know.

16. Can a square matrix with two identical rows be invertible? Explain.

17. Can a square matrix with two identical columns be invertible? Explain.

18. Let $\mathbf{A}$ and $\mathbf{B}$ be $n \times n$ matrices. Show that if $\mathbf{AB}$ is invertible, then so is $\mathbf{B}$.

19. Give the definition of a subspace of $\mathbf{R}^n$.

20. Let $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k \in \mathbf{R}^n$. Show that $\text{span}(\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k)$ is a subspace of $\mathbf{R}^n$.

21. Define the null space of a matrix. Show that the null space of an $n \times m$ matrix is a subspace of $\mathbf{R}^m$.

22. Define a basis for a subspace.

23. Determine if the vectors form a basis for $\mathbf{R}^2$.
$$\begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -2 \\ 5 \end{bmatrix}$$

24. Determine if the vectors form a basis for $\mathbf{R}^2$.
$$\begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -2 \\ -6 \end{bmatrix}$$

25. Determine if the vectors form a basis for $\mathbf{R}^3$.
$$\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ 7 \\ 0 \end{bmatrix}$$

26. Determine if the vectors form a basis for $\mathbf{R}^3$.
$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 4 \end{bmatrix}$$

27. Determine if the vectors form a basis for $\mathbf{R}^3$.
$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$$

28. Find the dimension of the span of the vectors
$$\begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 6 \end{bmatrix}.$$

29. Find the dimension of the span of the vectors
$$\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 7 \\ 1 \end{bmatrix}.$$

30. Let B be the basis
$$\begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$
Write the B coordinates of
$$\begin{bmatrix} 4 \\ 6 \end{bmatrix}.$$

31. Let B be the basis

$$\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$$

for a subspace H of $\mathbf{R}^3$ and let

$$\mathbf{v} = \begin{bmatrix} 5 \\ 4 \\ 9 \end{bmatrix}.$$

Compute $[\mathbf{v}]_B$.

32. Find a basis for the null space and the column space of the matrix

$$\begin{bmatrix} 1 & 1 & 5 \\ 0 & 1 & 3 \\ 2 & 3 & 13 \end{bmatrix}.$$

Give the dimensions of the null space and the column space.

33. If the rank of a 7×6 matrix $\mathbf{A}$ is 5, what is the dimension of the subspace of all solutions to $\mathbf{A}x = 0$?