1. Solve the system of linear equations by row reducing an appropriate augmented matrix. If the solution is not unique, then write your answer in parametric form.

$$\begin{array}{rcrcrcr} 4x & + & y & - & 3z & = & 3 \\ x & - & 5y & + & 7z & = & 3 \\ 8x & + & 2y & - & 10z & = & -6 \end{array}$$

2. Solve the matrix equation

$$\begin{bmatrix} 7 & 6 & -5 \\ 4 & -10 & 4 \\ 3 & 16 & -9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 8 \\ -8 \end{bmatrix}.$$

If the solution is not unique, then write your answer in parametric for.

3. Solve the system of equations by first reducing the augmented matrix to reduced row echelon form. If the solution is not unique, then write your answer in parametric for.

$$\begin{array}{rcrcrcr} 4x & + & y & + & 7z & = & 11 \\ 3x & + & y & + & 6z & = & 9 \\ 2x & + & y & + & 5z & = & 7 \end{array}$$

4. a) Give the definition of $\mathbf{span}(\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n)$.

   b) Is the vector $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ an element of $\mathbf{span}\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \end{bmatrix}\right)$? If so, find the coefficients (or weights). If not state how you know.

5. Is the vector $\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$ an element of $\mathbf{span}\left(\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 8 \\ 8 \end{bmatrix}\right)$? If so, find the coefficients (or weights). If not state how you know.

6. Explain why a vector $\mathbf{b}$ is in the span of vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ if and only if the matrix equation

$$\mathbf{Ax} = \mathbf{b}$$

is consistent where the columns of $\mathbf{A}$ are the vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$. Use the definition of span and consistent in your explanation.

7. Explain why the columns of an $n \times m$ matrix $\mathbf{A}$ span all of $\mathbf{R}^n$ if and only if when $\mathbf{A}$ is row reduced, every row has a nonzero leading entry.

8. Find the parametric form of the equation of the line through $\begin{bmatrix} 1 \\ -4 \\ 2 \end{bmatrix}$ and parallel with the vector $\begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}$.

9. Does the system

$$\begin{array}{rcrcrcr} x & + & 2y & + & 7z & = & 0 \\ 3x & + & y & + & 2z & = & 0 \\ 2x & - & y & - & 5z & = & 0 \end{array}$$

have a nontrivial solution? Show why or why not.

10. In which of the situations can you say if the vetors that make up the columns

of the matrix **A** are linearly independent or linearly dependent? Explain your answer.

a) **A** is 4×5.

b) **A** is 5×4.

c) Each row of **A** sums to zero.

d) Each column of **A** sums to zero.

e) When row reduced, each row has a pivot element.

f) When row redcued, each column has a pivot element.

11. a) Show that the vectors

$$\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

are linearly dependent.

b) Write one of the vectors in a) as a linear combination of the vectors that preceed it.

12. Let

$$S = \left\{ \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 6 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$$

a) Does the set S span $\mathbf{R}^3$? Justify your answer.

b) Is S a linearly independent set? Justify your answer.

13. Let

$$S = \left\{ \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 4 \end{bmatrix} \right\}$$

a) Does the set S span $\mathbf{R}^3$? Justify your answer.

b) Is S a linearly independent set? Justify your answer.

14. Let

$$S = \left\{ \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \right\}$$

a) Does the set S span $\mathbf{R}^3$? Justify your answer.

b) Is S a linearly independent set? Justify your answer.

15. Let

$$S = \left\{ \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \\ -2 \end{bmatrix} \right\}$$

a) Does the set S span $\mathbf{R}^3$? Justify your answer.

b) Is S a linearly independent set? Justify your answer.

16. Let

$$S = \left\{ \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ -1 \end{bmatrix} \right\}$$

a) Does the set S span $\mathbf{R}^3$? Justify your answer.

b) Is S a linearly independent set? Justify your answer.

17. Find all values of λ that makes the column vectors in the matrix

$$\begin{bmatrix} 1-\lambda & 0 \\ -1 & 2-\lambda \end{bmatrix}$$

linearly dependent.

18. Find all values of λ that makes the column vectors in the matrix

$$\begin{bmatrix} 5-\lambda & 2 \\ -24 & -9-\lambda \end{bmatrix}$$

linearly dependent.

19. Give the definitions:

    a) Linear transformation $T : \mathbf{R}^n \to \mathbf{R}^m$.

    b) The standard matrix for a linear transformation $T : \mathbf{R}^n \to \mathbf{R}^m$.

20. Let T be the linear transformation of the plane that reflects a vector about the x-axis and then rotates it by $\pi/2$. Find the standard matrix for T and use it to compute what T does to the vector $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$

21. Let T be the linear transformation in space that reflects a vector through the origin and then rotates it by $\pi/2$ around the y-axis. Find the standard matrix for T and use it to compute

$$T\left(\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}\right).$$

22. Let $\mathbf{A} = \begin{bmatrix} 1 & 3 \\ 2 & 9 \end{bmatrix}$ and $T : \mathbf{R}^2 \to \mathbf{R}^2$ be given by $T(\mathbf{v}) = \mathbf{A}\mathbf{v}$.

    a) Determine if T is one-to-one. Explain.

    b) Determine if T is onto. Explain.

23. Let

$$A = \begin{bmatrix} 1 & 2 & 1 & 3 \\ 2 & 5 & 4 & 7 \\ 1 & 3 & 3 & 4 \end{bmatrix}$$

and $T : \mathbf{R}^4 \to \mathbf{R}^3$ be given by $T(\mathbf{v}) = \mathbf{A}\mathbf{v}$.

    a) Determine if T is one-to-one. Explain.

    b) Determine if T is onto. Explain.

24. Let

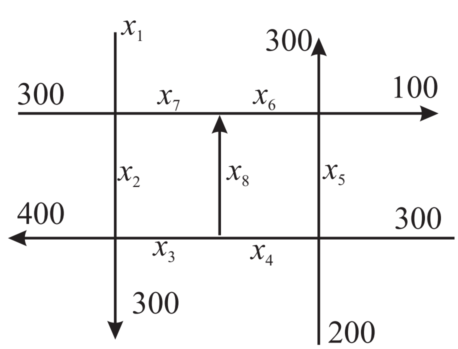
$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 5 & 3 \\ 1 & 4 & 3 \\ 3 & 7 & 4 \end{bmatrix}$$

and $T : \mathbf{R}^3 \to \mathbf{R}^4$ be given by $T(\mathbf{v}) = \mathbf{A}\mathbf{v}$.

    a) Determine if T is one-to-one. Explain.

    b) Determine if T is onto. Explain.

25. Set up the equations and solve for the traffic flow problem in the diagram. Find the range of flows for each variable.



26. The population projection matrix for a species of birds is

$$\begin{bmatrix} .7 & 1.1 \\ .9 & .8 \end{bmatrix}.$$

The year 1 populations of hatchlings and adults are 100 and 200, respectively.

a) Project the populations in year 2.

b) Project the populations in year 3.

c) Project the population in year 0. (Solve mathematically and decide if the answer makes sense.)