Group Project Rules and Encouragement

First, this is a major, lengthy assignment. You should start today and work on it some every day until you complete the project. Your paper will be due at the start of class on April 25.

1. If you have not already done so decide who will be in your group and plan to meet as a group today or tomorrow.
2. Start today. If you have the problem well in mind, your subconscious will work for you. You may find that good ideas will come to you at the strangest times.
3. Today, start by reading the entire project so you can see what the big picture is. Don't worry about details at first. Do this before the group meets so everyone has the overall picture in mind. You should probably discuss the project to be sure everyone has it well in mind and everyone agrees what the project is saying.
4. Next read the project in detail. Begin to think about the details you are to work out. At some point you may hit a snag. At that point you will zero in on the obstacles. Keep thinking about the obstacles so that you will have them well enough in mind to be able to work on the problems any time you have a spare minute or two.
5. You are to work as a group. You should discuss your progress as a group and help each other through the tough spots. You may discuss your progress on the project in general terms with friends outside your group, but do **not** give hints or help to other groups. Your group is not alone in this project. If you hit a snag and put substantial effort into the project with no progress, **one person** from your group should go to your teacher's office and ask for a hint. Only one person from the group is to go to your teacher's office at a time. That person will then be responsible to communicate to the others in the group what was said. One of the objectives of a group project is to give you practice in communicating mathematical ideas to others, so if your group seeks help more than once, take turns going to your teacher's office.
7. **Do not go to anyone but your teacher for help on the project. This includes other people, books, the web, or any other source of information. The purpose is not to reproduce what someone else has already done, but to figure it out yourself.**
8. When you have completed the work on the project it is time to prepare it in written form. Your group will prepare only one report, not one for each member of the group. Your paper should be a mix of equations, formulas, and prose to support your conclusions. Pictures are also sometimes helpful. Use complete sentences, good grammar and correct punctuation. Spelling is also important. The prose is written to convey to the reader an explanation of what you have done. It should be written in such a way that an A student in another section of calculus would be able to understand it without the project directions. You will be graded on your written presentation as well as the mathematical content. Be sure each member of the group understands everything that was done on the project. You may have to meet a few times after the report is written to be sure everyone knows all the details.
9. After you have completed the project think about how much work each member of your group did on the project. Each person in the group is to turn in the Group Evaluation Form. Your response to the form will be seen only by you and your instructor. For some of the groups a grade may not be assigned until after an interview with your teacher. Your individual grade will usually be determined by the written report only. On rare occasions, when someone did considerably less than his or her share, an individual's grade will be determined by the report, how much of the project he or she did and in certain cases an interview. In most cases the letter grade for each member of the group is the same.

By signing below, you are stating that you have followed the project rules.

Name Date

Group Evaluation Form

Name

1. List the names of the other members of your group. Next to each name, write a few words indicating how that person contributed and your best estimate of what percent of the project was done by this is person (include yourself). (For example, Sue - typed the report and solved part 2 - 25%)

2. Did some in your group show special insight on how to solve parts of the problem? If so, please indicate who and describe what they did.

3. Did some in your group emerge as leaders? If so, please indicate who and describe in what way they were leaders.

4. Did the group work well together? Explain.

5. Do you prefer group projects or individual projects?

A Probability Computation.

Tom rolled a fair die 6 times and found that the number 3 came up twice while the number 2 did not come up at all. Tom thought that since each number is equally likely, on average each number should come up exactly once out of 6 rolls. Therefore, it seemed to him that it would be likely that out of six rolls of the die each number should come up exactly once. His sister Mary, who was studying probability at the time, told him that it was not likely because even if after the first 5 rolls all 5 numbers were different, the probability of getting the remaining number on roll 6 was only $\frac{1}{6}$.

After thinking about it for a day or two, Tom agreed with Mary. Then he wondered what would happen if the die had more than 6 sides. He asked Mary what she thought the probability would be after tossing a die with n sides n times that each side would come up exactly once where n is a very large number. Mary thought that the answer would be about $\left(\frac{1}{2}\right)^n$. She explained that for each of the first $\frac{n}{2}$ tosses the probability that toss yields an outcome different from all the previous tosses is more than $\frac{1}{2}$ while for each of the last $\frac{n}{2}$ tosses the probability would be less than $\frac{1}{2}$. So, she explained, on the average it would be like flipping a fair coin n times and computing the probability that a head comes up each time. Tom, not wanting to appear completely at a loss (regardless of the facts) said he thought the probability was less than $\left(\frac{1}{2}\right)^n$.

It is your job to settle the argument. But before you begin, there are a few useful formulas which you should be aware of.

Given the numbers $1, 2, 3, \cdots, n$ there are exactly $n!$ ways of arranging the numbers in a row. For example, if $n = 3$ then the $3! = 6$ ways of arranging the numbers in a row are 123, 132, 213, 231, 312, and 321.

If you do an experiment n times and each time you do the experiment there are k possible outcomes for that experiment, then there are a total of k^n different sequences of outcomes from the n experiments. For example, if you flip a coin 3 times there are $2^3 = 8$ possible outcomes: HHH, HHT, HTH, HTT, THH, THT, TTH, TTT.

If every outcome among n possible outcomes is equally likely and you want to know the probability that an outcome is one of a group of say k outcomes then the probability is $\frac{k}{n}$. For example, when you flip a coin once the probability of a head is $\frac{1}{2}$ since there are two possible outcomes and only one is a head. On the other hand, the probability of getting exactly two heads when a fair coin is flipped three times is $\frac{3}{8}$ since there are eight possible outcomes (as listed above) and of these three have two heads.

Now follow the steps below to settle the argument.

1. How many different possible sequences of outcomes are possible when a die with n sides is tossed n times?
2. In how many of the sequences in part 1) do each of the n possible outcomes occur exactly once?
3. Based on 1) and 2) what is the probability that among n tosses of an n–sided die each toss yields a different number from each of the others? (Just write your answer as a fraction involving the parameter n.)
4. In order to settle the argument you are to compute the n^{th} root of the number you wrote in part 3). Write this formula and simplify.
5. Now compute the limit of the expression you found in part 4). This is the most significant part of the project. You may wish to follow the hints below:
   a. It will simplify things to compute the limit of the natural log.
   b. After taking log, simplify and make your expression look like a Riemann sum. You may need to review what a Riemann sum is. You should get an integral of some function evaluated between 0 and 1.
   c. If the Reimann sum gives an improper integral then compute it by computing the appropriate limits.
6. Based on your computations, who is right? Why?