1. Determine if the series converges or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} 1.000001^n$$

2. Determine if the series converges or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n} + n + \sqrt{n}}$$

3. Determine if the series converges or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} \frac{1}{3n+7}$$

4. Determine if the series converges or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} \frac{n^2 + 2n + 1}{10n^3 + 3n^2 + 4n - 1}$$

5. Determine if the series converges or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} (2n!)e^{-n/2}$$

6. Determine if the series converges or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} \frac{3}{\sqrt{n^2 + n + 3}}$$

7. Determine if the series converges or diverges. Justify carefully.  $\sum_{n=1}^{\infty} \left(1 - \frac{1}{n}\right)^{n^2}$ 

8. Determine if the series converges or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n+1)!}$$

9. Determine if the series converges or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} \tan(\frac{1}{n^2})$$

10. Determine if the series converges absolutely, converges, or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} (-1)^n \frac{n^3}{3^n}$$

11. Determine if the series converges absolutely, converges, or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} (-1)^n (\sqrt[3]{n+1} - \sqrt[3]{n})$$

12. Determine if the series converges absolutely, converges, or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} \frac{\cos(\frac{n\pi}{2})}{n}$$

- 13. Determine if the series converges absolutely, converges, or diverges. Justify carefully.  $\sum_{n=1}^{\infty} \left(\frac{1}{n} 1\right)^n$
- 14. Determine if the series converges absolutely, converges, or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$$

15. How many terms do you need to add to get a number that is within 0.0001 of the infinite sum for

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(4n+3)^3}$$

16. How many terms do you need to add to get a number that is within 0.01 of the infinite sum for

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n^2 + 7}$$

17. How many terms do you need to add to get a number that is within 0.0001 of the infinite sum for

$$\sum_{n=0}^{\infty} \frac{1}{3n+1}$$

Explain your answer.

18. Find the interval of convergence and the radius of convergence for

$$\sum_{n=0}^{\infty} \frac{\sqrt{n}x^n}{5^n}$$

19. Find the interval of convergence and the radius of convergence for

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$$

20. Find the interval of convergence and the radius of convergence for

$$\sum_{n=0}^{\infty} n^2 x^n$$

21. Find the interval of convergence and the radius of convergence for

$$\sum_{n=0}^{\infty} \frac{(x-2)^n}{n+1}$$

22. Find the interval of convergence and the radius of convergence for

$$\sum_{n=0}^{\infty} n! x^n$$

23. Find the interval of convergence and the radius of convergence for

$$\sum_{n=0}^{\infty} \frac{2n+1}{2^n+3^n} (x-1)^n$$

- 24. Find the power series for the function  $\sqrt[3]{(1+x)^4}$  expanded about x = 0.
- 25. Find the power series for the function  $\sqrt[5]{(1+x)^6}$  expanded about x = 0.
- 26. Find the power series for the function  $\left(\frac{1}{x+2}\right)^3$  expanded about x = 0.
- 27. Use the error term for the the Maclaurin series for  $e^x$  to estimate how close  $1+x+x^2/2$  is to  $e^x$  if  $-0.1 \le x \le 0.1$ .
- 28. What degree Maclaurin polynomial does one need in order to approximate  $\cos x$ to within 0.00005 for  $0 \le x \le 0.5$ ?
- 29. What degree Maclaurin polynomial does one need in order to approximate  $e^x$  to within 0.000005 for  $-1 \le x \le 1$ ?

30. Compute the derivative of the power series and compare the intervals of convergence of the series and its derivative.

$$\sum_{n=1}^{\infty} \frac{(x-2)^n}{n}$$

31. Compute the derivative of the power series and compare the intervals of convergence of the series and its derivative.

$$\sum_{n=0}^{\infty} \frac{x^n}{n!}$$

- 32. Find the third degree Maclaurin polynomial for the function  $f(x) = \tan x$ .
- 33. Find the fourth degree Taylor polynomial for the function  $\ln x$  expanded about x = 1.
- 34. Derive the Taylor series for  $\sin x$  expanded about x = 0.
- 35. Derive the Maclaurin series for  $e^x$ .
- 36. Use the error term for the the Maclaurin series for  $e^x$  to estimate how close  $1+x+x^2/2$  is to  $e^x$  if  $-0.1 \le x \le 0.1$ .
- 37. What degree Maclaurin polynomial does one need in order to approximate  $\cos x$ to within 0.00005 for  $0 \le x \le 0.5$ ?
- 38. What degree Maclaurin polynomial does one need in order to approximate  $e^x$  to within 0.000005 for  $-1 \le x \le 1$ ?

39. Derive the power series for  $\arctan x$  (expand about x = 0). Use this to compute the sum of the series

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$$

40. Derive the Maclaurin series for  $\ln(1+x)$ . Use this to compute the sum of the series

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$$
41. Compute  $\int x \sin(3x) dx$ 
42. Compute  $\int (x^2 + 3x - 1)e^{2x} dx$ 
43. Compute  $\int \sec(3x) dx$ 
44. Compute  $\int \sec(3x) dx$ 
45. Compute  $\int x^2 \ln(x) dx$ 
46. Compute  $\int \frac{3x^2 + 4x - 4}{x^3 + 2x^2 - 4x + 1} dx$ 
47. Compute  $\int \frac{5x - 9}{x^2 - 4x + 3} dx$ 
48. Compute  $\int \frac{x^2 + 9x + 15}{(x+2)(x^2 + 4x + 5)} dx$ 
49. Compute  $\int \frac{5x^2 + x + 9}{(x-1)^2(x+2)} dx$ 
50. Compute  $\int e^{2x} \cos(3x) dx$ 
51. Compute  $\int (x-1)\sqrt{2x+3} dx$