

1. Compute the following:

$$\frac{d}{dx} e^x \ln(x)$$

2. Compute the following:

$$\frac{d}{dx} x^2 \arcsin x$$

3. Compute the following:

$$\frac{d}{dx} \frac{\arctan(x)}{1+x^2}$$

4. Compute the following:

$$\frac{d}{dx} \log_2(2x^2)$$

5. Compute the following:

$$\frac{d}{dx} \log_x 10$$

6. Compute the following:

$$\frac{d}{dx} 3^{2x+4}$$

7. Compute the following:

$$\frac{d}{dx} \sinh(x^2 + 1)$$

8. Compute the following:

$$\frac{d}{dx} (\sinh^2 x \cosh(2x))$$

9. Compute the following:

$$\frac{d}{dx} \frac{x}{\sinh^{-1} x} \quad (\text{This problem refers to the inverse hyperbolic sine function.})$$

10. Compute the following:

$$\frac{d}{dx} \tanh^3(3x^2 + 2x - 1)$$

11. Derive the formula for the derivative of the inverse hyperbolic tangent function.

12. Compute the following:

$$\frac{d}{dx} e^{x^2+3x-5}$$

13. Compute the following:

$$\frac{d}{dx} (x \operatorname{arcsec}(x^2/2))$$

14. Compute the derivatives and explain why you use the differentiation rules that you do.

a)  $\frac{d}{dx} \sqrt{2^x}$

b)  $\frac{d}{dx} x^{\sqrt{2}}$

15. Compute:
- $\lim_{x \rightarrow 0} \frac{x \sin(x)}{1 - \cos(x)}$

16. Compute:
- $\lim_{x \rightarrow 0} \frac{\sin(x) - x + \frac{1}{6}x^3}{x^5}$

17. Compute:
- $\lim_{x \rightarrow 0^+} x^{-1/\ln x}$

18. Compute:
- $\lim_{x \rightarrow 0} \left( \frac{4x+1}{x} - \frac{1}{\sin x} \right)$

19. Compute:
- $\lim_{x \rightarrow \infty} \frac{1}{x \ln x} \int_1^x \ln t \, dt$

20. Compute the following:

$$\int_{\pi/8}^{\pi/6} \tan(2x) \, dx$$

21. Compute the following:

$$\int \frac{3 \ln x^2}{x} \, dx$$

22. Compute the following:

$$\int \frac{1}{x \log_2 x} dx$$

23. Compute:  $\int \cot(x) dx$

24. Compute the following:

$$\int \frac{\sec^2(3x)}{4 + \tan(3x)} dx$$

25. Compute the following:

$$\int \frac{e^x}{1 + e^x} dx$$

26. Compute the following:

$$\int_{-\frac{1}{2} \ln 3}^0 \left( \frac{e^x}{1 + e^{2x}} \right) dx$$

27. Compute the following:

$$\int 3^x dx$$

28. Compute the following:

$$\int \sin(x) e^{\cos(x)} dx$$

29. Compute the following:

$$\int \frac{1}{\sqrt{9 - 4x^2}} dx$$

30. Compute the following:

$$\int \frac{1}{2 + x^2} dx$$

31. Compute the following:

$$\int x^{3x^2} dx$$

32. Compute the following:

$$\int \frac{1}{x\sqrt{4x^2 - 9}} dx$$

33. Compute the following:

$$\int \frac{1}{1 + 16x^2} dx$$

34. Compute the following:

$$\int \frac{1}{x\sqrt{4x^2 - 1}} dx$$

35. Compute the following:

$$\int \frac{\arcsin(x)}{\sqrt{1 - x^2}} dx$$

36. Compute the following:

$$\int \frac{1}{3x + 2} dx$$

37. Compute  $\int \frac{3}{9 - t^2} dt$

38. Compute  $\int \frac{1}{t\sqrt{25 - t^2}} dt$

39. Compute the following:

$$\frac{d}{dx} \int_{e^{-x}}^{e^x} \ln t dt$$

40. Compute two different ways:

$$\frac{d}{dx} x^{2x}$$

41. Compute the following:

$$\frac{d}{dx} \log_x(x + 1)$$

42. Compute using logarithmic differentiation.

$$\frac{d}{dx} \sqrt[4]{\frac{(x^2 + 1)x^3}{(2x - 1)^3 \sqrt{x - 1}}}$$

43. Find an equation of the line tangent to  $y = \arctan x$  at the point where  $x = 1$ .

44. Find an equation of the line tangent to the graph of  $y = xe^{x^2}$  at the point where  $x = 2$ .
45. Find an equation for the line tangent to  $y = x \ln x$  at the point where  $x = e$ .
46. Find the area bounded by the graphs of  $y = e^x$  and  $y = (e - 1)x + 1$ .
47. Find the volume of the solid of revolution obtained by rotating the region bounded by the coordinate axes,  $y = 3$ , and  $x = \frac{2}{\sqrt{y+1}}$  about the  $y$ -axes.
48. Find the absolute maximum and minimum values of the function  $f(x) = e^x - 2x$  for  $-1 \leq x \leq 2$ .
49. Show that any curve of the form  $y = -\frac{1}{2}x^2 + k$  and any curve of the form  $y = \ln(x) + c$  intersect each other at right angles.
50. Show that  $\cosh^{-1} x = \ln(x + \sqrt{x^2 - 1})$  as long as  $x \geq 1$ .
51. Using the definition of  $\ln$ , prove that for any positive real numbers  $a$  and  $b$ ,  $\ln(ab) = \ln(a) + \ln(b)$ .
52. Using the definition of  $\ln$ , prove that  $\ln x$  is an increasing function.
53. Using the definition of the exponential function  $\text{Exp}$ , derive the formula for the derivative of  $\text{Exp}(x)$ .
54. Derive the formula for  $\frac{d}{dx} \arctan x$ .
55. Derive the formula for  $\frac{d}{dx} \arccos x$ .
56. Use the definition of  $\log_a x$  and properties of  $\ln$ , prove that for any positive  $a, b, c$ ,  $\log_a(bc) = \log_a(b) + \log_a(c)$ .
57. Use the definition of the exponential function  $\text{Exp}(x)$ , prove that  $\text{Exp}(x + y) = \text{Exp}(x)\text{Exp}(y)$ .
58. Let  $a > 0$ . Give the definition of  $a^x$  in terms of the exponential function. Use this definition (and properties of the exponential function) to show that  $a^{x+y} = a^x a^y$  for any real numbers  $x$  and  $y$ .
59. Give the domain and range for each of the inverse trigonometric functions.
60. Prove that  $\arcsin x + \arccos x = \frac{\pi}{2}$ .
61. How many zeros does the function  $f(x) = 2^x - 1 - x^2$  have on the real line? (Prove your answer.)
62. Find the formula for the  $n^{\text{th}}$  term of the sequence that starts
- $$1, 1, 3, 3, 5, 5, 7, 7, \dots$$
63. Find the formula for the  $n^{\text{th}}$  term of the sequence that starts
- $$0, 7, 0, 7, 0, 7, \dots$$
64. Find the formula for the  $n^{\text{th}}$  term of the sequence that starts
- $$1, \frac{1}{5}, \frac{1}{25}, \frac{1}{125}, \dots$$

65. Does the sequence below converge? If so find its limit.

$$a_n = \sqrt{\frac{2n+3}{3n-1}}$$

66. Does the sequence below converge? If so find its limit.

$$a_n = \frac{3n^2 + 2n - 1}{1 + 5n + 7n^2}$$

67. Does the sequence below converge? If so find its limit.

$$a_n = (-1)^n + 1$$

68. Does the sequence below converge? If so find its limit.

$$b_n = \frac{n!}{n^n}$$

69. Does the sequence below converge? If so find its limit.

$$c_n = \sqrt[n]{n+1}$$

70. Determine if the sequence below is bounded above, bounded below, nondecreasing, or nonincreasing.

$$a_n = \frac{n+1}{2n+3}$$

71. Determine if the sequence below is bounded above, bounded below, nondecreasing, or nonincreasing.

$$a_n = \sin n + \frac{1}{n}$$

72. If 10g of a radioactive substance decays to 8g in 3 years, find its half life. If the half life of another substance is 1 year, find how long it takes 10g of the substance to decay to 8g.

73. Suppose that after 10 years of earning interest at the rate of 7% per year, you have \$100,000 in an account. How much money was invested in this account 10 years ago to earn this amount? (No money was added in the mean time.)

74. A rectangle with sides parallel to the axis is to have one corner at the origin and the other on the curve  $y = e^{-x^2}$ . What is the largest rectangular area that can be formed in this way?

75. In a solution, the product of the hydronium ion concentration  $[\text{H}_3\text{O}^+]$  and the hydroxyl ion concentration  $[\text{OH}^-]$  (both in moles per liter) is  $10^{-14}$ .

- What value of  $[\text{H}_3\text{O}^+]$  minimizes the sum  $S = [\text{OH}^-] + [\text{H}_3\text{O}^+]$ ?
- What is the pH of solution you found in part a)?
- What ratio of  $[\text{H}_3\text{O}^+]$  to  $[\text{OH}^-]$  minimizes  $S$ ?

76. Compute

$$\sum_{n=0}^{\infty} \frac{\pi^n}{4^n}$$

77. Compute

$$\sum_{n=0}^{\infty} (-1)^n \frac{1}{5^n}$$

78. Compute

$$\sum_{n=1}^{\infty} \frac{4}{(4n-3)(4n+1)}$$

79. Determine if the series converges or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} 1.000001^n$$

80. Compute and state for which values of  $x$  the series converges

$$\sum_{n=0}^{\infty} e^{nx}$$

81. Compute

$$\sum_{n=1}^{\infty} \frac{1}{4n^2 - 1}$$

82. Write the number  $2.373737\dots$  as a ratio of two integers.

83. Assume that  $|x| < 1$  and compute the sum

$$\sum_{n=0}^{\infty} x^n.$$

Use your answer to compute the sum

$$\sum_{n=1}^{\infty} nx^{n-1}.$$