

1. Find  $\lim_{x \rightarrow 0} \frac{e^x - 1}{\sin x}$

2. Find  $\lim_{x \rightarrow 0} \frac{\cos x - 1 - x^2/2}{x^4}$

3. Find  $\lim_{n \rightarrow \infty} \left(1 - \frac{1}{n}\right)^{2n}$

4. Find  $\lim_{n \rightarrow \infty} n^{\frac{1}{n}}$

5. Find  $\lim_{n \rightarrow \infty} \frac{n^2}{1.00001^n}$

6. Find  $\lim_{t \rightarrow 0^+} t \ln t$

7. Find the formula for the  $n^{\text{th}}$  term of the sequence that starts

$$1, 1, 3, 3, 5, 5, 7, 7, \dots$$

8. Find the formula for the  $n^{\text{th}}$  term of the sequence that starts

$$0, 7, 0, 7, 0, 7, \dots$$

9. Find the formula for the  $n^{\text{th}}$  term of the sequence that starts

$$1, \frac{1}{5}, \frac{1}{25}, \frac{1}{125}, \dots$$

10. Does the sequence below converge? If so find its limit.

$$a_n = \sqrt{\frac{2n+3}{3n-1}}$$

11. Does the sequence below converge? If so find its limit.

$$a_n = \frac{3n^2 + 2n - 1}{1 + 5n + 7n^2}$$

12. Does the sequence below converge? If so find its limit.

$$a_n = (-1)^n + 1$$

13. Does the sequence below converge? If so find its limit.

$$b_n = \frac{n!}{n^n}$$

14. Does the sequence below converge? If so find its limit.

$$c_n = \sqrt[n]{n+1}$$

15. Find the limit of the sequence and prove it is what you say it is.

$$a_n = \frac{n}{n+1}$$

16. Find the limit of the sequence and prove it is what you say it is.

$$a_n = \frac{\sin n}{n}$$

17. Find the limit of the sequence and prove it is what you say it is.

$$a_n = \frac{\sqrt{n}}{n+1}$$

18. Determine if the sequence below is bounded above, bounded below, nondecreasing, or nonincreasing.

$$a_n = \frac{n+1}{2n+3}$$

19. Determine if the sequence below is bounded above, bounded below, nondecreasing, or nonincreasing.

$$a_n = \sin n + \frac{1}{n}$$

20. Compute

$$\sum_{n=0}^{\infty} \frac{\pi^n}{4^n}$$

21. Compute

$$\sum_{n=0}^{\infty} (-1)^n \frac{1}{5^n}$$

22. Compute

$$\sum_{n=1}^{\infty} \frac{4}{(4n-3)(4n+1)}$$

23. Determine if the series converges or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} 1.000001^n$$

24. Determine if the series converges or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n} + n + \sqrt{n}}$$

25. Determine if the series converges or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} \frac{1}{3n+7}$$

26. Determine if the series converges or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} \frac{n^2 + 2n + 1}{10n^3 + 3n^2 + 4n - 1}$$

27. Determine if the series converges or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} (2n!)e^{-n/2}$$

28. Determine if the series converges or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} \frac{3}{\sqrt{n^2 + n + 3}}$$

29. Determine if the series converges or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n+1)!}$$

30. Determine if the series converges or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} \tan\left(\frac{1}{n^2}\right)$$

24. Determine if the series converges or diverges. Justify carefully.

31. Determine if the series converges absolutely, converges, or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} (-1)^n \frac{n^3}{3^n}$$

32. Determine if the series converges absolutely, converges, or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} (-1)^n (\sqrt[3]{n+1} - \sqrt[3]{n})$$

33. Determine if the series converges absolutely, converges, or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} \frac{\cos\left(\frac{n\pi}{2}\right)}{n}$$

34. Determine if the series converges absolutely, converges, or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$$

35. How many terms do you need to add to get a number that is within 0.0001 of the infinite sum for

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(4n+3)^3}$$

36. How many terms do you need to add to get a number that is within 0.01 of the infinite sum for

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n^2+7}$$

37. How many terms do you need to add to get a number that is within 0.0001 of the infinite sum for

$$\sum_{n=0}^{\infty} \frac{1}{3n+1}$$

Explain your answer.

38. Find the interval of convergence and the radius of convergence for

$$\sum_{n=0}^{\infty} \frac{\sqrt{n}x^n}{5^n}$$

39. Find the interval of convergence and the radius of convergence for

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$$

40. Find the interval of convergence and the radius of convergence for

$$\sum_{n=0}^{\infty} n^2 x^n$$

41. Find the interval of convergence and the radius of convergence for

$$\sum_{n=0}^{\infty} \frac{(x-2)^n}{n+1}$$

42. Find the interval of convergence and the radius of convergence for

$$\sum_{n=0}^{\infty} n! x^n$$

43. Find the interval of convergence and the radius of convergence for

$$\sum_{n=0}^{\infty} \frac{2n+1}{2^n+3^n} (x-1)^n$$

44. Compute the derivative of the power series and compare the intervals of convergence of the series and its derivative.

$$\sum_{n=1}^{\infty} \frac{(x-2)^n}{n}$$

45. Compute the derivative of the power series and compare the intervals of convergence of the series and its derivative.

$$\sum_{n=0}^{\infty} \frac{x^n}{n!}$$

46. Find the power series for the function  $\sqrt[3]{(1+x)^4}$  expanded about  $x = 0$ .

47. Find the power series for the function  $\sqrt[5]{(1+x)^6}$  expanded about  $x = 0$ .

48. Find the power series for the function  $\left(\frac{1}{x+2}\right)^3$  expanded about  $x = 0$ .

49. Find the third degree Maclaurin polynomial for the function  $f(x) = \tan x$ .

50. Find the fourth degree Taylor polynomial for the function  $\ln x$  expanded about  $x = 1$ .

51. Derive the Taylor series for  $\sin x$  expanded about  $x = 0$ .

52. Derive the Maclaurin series for  $e^x$ .

53. Use the error term for the the Maclaurin series for  $e^x$  to estimate how close  $1+x+x^2/2$  is to  $e^x$  if  $-0.1 \leq x \leq 0.1$ .

54. What degree Maclaurin polynomial does one need in order to approximate  $\cos x$  to within 0.00005 for  $0 \leq x \leq 0.5$ ?

55. What degree Maclaurin polynomial does one need in order to approximate  $e^x$  to within 0.000005 for  $-1 \leq x \leq 1$ ?

56. Derive the power series for  $\arctan x$  (expand about  $x = 0$ ). Use this to compute the sum of the series

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$$

57. Derive the Maclaurin series for  $\ln(1+x)$ . Use this to compute the sum of the series

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$$