

1. Compute the following:

$$\frac{d}{dx}x^{(x^x)}$$

2. Compute the following:

$$\frac{d}{dx}(x^x)^x$$

3. Solve the differential equation:

$$y'x = \frac{2y}{x}, x > 0$$

4. Solve the differential equation:

$$xy' + 2y = \frac{2}{x}, x > 0.$$

5. Solve the differential equation:

$$x^2y' = e^y, x > 0$$

6. Solve the differential equation:

$$xy' = x^2 - y, x > 0$$

7. Solve the differential equation:

$$y' + \frac{y}{(1+x^2)\arctan(x)} = \frac{x}{\arctan(x)}, x > 0.$$

8. Solve the initial value problem:

$$y' + y \cot x = 1, y\left(\frac{\pi}{2}\right) = 1$$

9. solve the initial value problem:

$$y' = 3e^{x+y}, y(0) = -1$$

10. If 10g of a radioactive substance decays to 8g in 3 years, find its half life. If the half life of another substance is 1 year, find how long it takes 10g of the substance to decay to 8g.

11. Suppose that after 10 years of earning interest at the rate of 7% per year, you have \$100,000 in an account. How much money was invested in this account 10 years ago to earn this amount? (No money was added in the mean time.)

12. A rectangle with sides parallel to the axis is to have one corner at the origin and the other on the curve  $y = e^{-x^2}$ . What is the largest rectangular area that can be formed in this way?

13. In a solution, the product of the hydronium ion concentration  $[\text{H}_3\text{O}^+]$  and the hydroxyl ion concentration  $[\text{OH}^-]$  (both in moles per liter) is  $10^{-14}$ .

- What value of  $[\text{H}_3\text{O}^+]$  minimizes the sum  $S = [\text{OH}^-] + [\text{H}_3\text{O}^+]$ ?
- What is the pH of solution you found in part a)?
- What ratio of  $[\text{H}_3\text{O}^+]$  to  $[\text{OH}^-]$  minimizes  $S$ ?

14. Do the first two steps of Euler's method for the initial value problem

$$y' = x + y \quad y(0) = 1.$$

Assume you are solving it for  $0 \leq x \leq 10$  and  $n = 10$ .

15. A 5 liter container is filled with a solution of salt having concentration 0.5 g/cc. A solution with concentration 2.5 g/cc of salt is added to the 5 liter container at the rate of 100 cc/hour while the the salt solution is drained from the 5 liter solution at the same rate, leaving 5 liters of solution in the container at all times. The solution is well mixed. Find a function that gives the concentration of salt in the 5 liter solution at all times  $t$ .

16. The region in the plane bounded by  $x = 1$ ,  $x = 2$ , the  $x$ -axis, and the graph of  $f(x) = \frac{1}{x(1+x^2)}$  is rotated about the  $y$ -axis. Find the volume generated.

17. Derive the formula  $\frac{d}{dx} \sinh x = \cosh x$

18. Derive the formula for the inverse hyperbolic cosine function.

19. Compute the following:

$$\frac{d}{dx} \frac{\sinh x + \cosh(2x + 1)}{x + 1}$$