

1. Compute

$$\sum_{n=0}^{\infty} \frac{\pi^n}{4^n}$$

2. Compute

$$\sum_{n=0}^{\infty} (-1)^n \frac{1}{5^n}$$

3. Compute

$$\sum_{n=1}^{\infty} \frac{4}{(4n-3)(4n+1)}$$

4. Determine if the series converges or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} 1.000001^n$$

5. Determine if the series converges or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n} + n + \sqrt{n}}$$

6. Determine if the series converges or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} \frac{1}{3n+7}$$

7. Determine if the series converges or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} \frac{n^2 + 2n + 1}{10n^3 + 3n^2 + 4n - 1}$$

8. Determine if the series converges or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} (2n!)e^{-n/2}$$

9. Determine if the series converges or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} \frac{3}{\sqrt{n^2 + n + 3}}$$

10. Determine if the series converges or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n+1)!}$$

11. Determine if the series converges or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} \tan\left(\frac{1}{n^2}\right)$$

12. Determine if the series converges absolutely, converges, or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} (-1)^n \frac{n^3}{3^n}$$

13. Determine if the series converges absolutely, converges, or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} (-1)^n (\sqrt[3]{n+1} - \sqrt[3]{n})$$

14. Determine if the series converges absolutely, converges, or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} \frac{\cos\left(\frac{n\pi}{2}\right)}{n}$$

15. Determine if the series converges absolutely, converges, or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$$

16. How many terms do you need to add to get a number that is within 0.0001 of the infinite sum for

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(4n+3)^3}$$

17. How many terms do you need to add to get a number that is within 0.01 of the infinite sum for

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n^2+7}$$

18. How many terms do you need to add to get a number that is within 0.0001 of the infinite sum for

$$\sum_{n=0}^{\infty} \frac{1}{3n+1}$$

Explain your answer.

19. Find the interval of convergence and the radius of convergence for

$$\sum_{n=0}^{\infty} \frac{\sqrt{n}x^n}{5^n}$$

20. Find the interval of convergence and the radius of convergence for

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$$

21. Find the interval of convergence and the radius of convergence for

$$\sum_{n=0}^{\infty} n^2 x^n$$

22. Find the interval of convergence and the radius of convergence for

$$\sum_{n=0}^{\infty} \frac{(x-2)^n}{n+1}$$

23. Find the interval of convergence and the radius of convergence for

$$\sum_{n=0}^{\infty} n! x^n$$

24. Find the interval of convergence and the radius of convergence for

$$\sum_{n=0}^{\infty} \frac{2n+1}{2^n+3^n} (x-1)^n$$

25. Compute the derivative of the power series and compare the intervals of convergence of the series and its derivative.

$$\sum_{n=1}^{\infty} \frac{(x-2)^n}{n}$$

26. Compute the derivative of the power series and compare the intervals of convergence of the series and its derivative.

$$\sum_{n=0}^{\infty} \frac{x^n}{n!}$$

27. Find the power series for the function  $\sqrt[3]{(1+x)^4}$  expanded about  $x=0$ .

28. Find the power series for the function  $\sqrt[5]{(1+x)^6}$  expanded about  $x = 0$ .
29. Find the power series for the function  $\left(\frac{1}{x+2}\right)^3$  expanded about  $x = 0$ .
30. Find the third degree Maclaurin polynomial for the function  $f(x) = \tan x$ .
31. Find the fourth degree Taylor polynomial for the function  $\ln x$  expanded about  $x = 1$ .
32. Devire the Taylor series for  $\sin x$  expanded about  $x = 0$ .
33. Derive the Maclaurin series for  $e^x$ .
34. Use the error term for the the Maclaurin series for  $e^x$  to estimate how close  $1+x+x^2/2$  is to  $e^x$  if  $-0.1 \leq x \leq 0.1$ .
35. What degree Maclaurin polynomial does one need in order to approximate  $\cos x$  to within 0.00005 for  $0 \leq x \leq 0.5$ ?
36. What degree Maclaurin polynomial does one need in order to approximate  $e^x$  to within 0.000005 for  $-1 \leq x \leq 1$ ?
37. Derive the power series for  $\tan x$  (expand about  $x = 0$ ). Use this to compute the sum of the series

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$$

38. Derive the Maclaurin series for  $\ln(1+x)$ . Use this to compute the sum of the series

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$$