1. Compute \[ \sum_{n=0}^{\infty} \frac{\pi^n}{4^n} \]

2. Compute \[ \sum_{n=0}^{\infty} \frac{(-1)^n}{5^n} \]

3. Compute \[ \sum_{n=1}^{\infty} \frac{4}{(4n - 3)(4n + 1)} \]

4. Determine if the series converges or diverges. Justify carefully.
\[ \sum_{n=1}^{\infty} 1.000001^n \]

5. Determine if the series converges or diverges. Justify carefully.
\[ \sum_{n=1}^{\infty} \frac{1}{n\sqrt{n} + n + \sqrt{n}} \]

6. Determine if the series converges or diverges. Justify carefully.
\[ \sum_{n=1}^{\infty} \frac{1}{3n + 7} \]

7. Determine if the series converges or diverges. Justify carefully.
\[ \sum_{n=1}^{\infty} \frac{n^2 + 2n + 1}{10n^3 + 3n^2 + 4n - 1} \]

8. Determine if the series converges or diverges. Justify carefully.
\[ \sum_{n=1}^{\infty} (2n!)e^{-n/2} \]

9. Determine if the series converges or diverges. Justify carefully.
\[ \sum_{n=1}^{\infty} \frac{3}{\sqrt{n^2 + n + 3}} \]

10. Determine if the series converges or diverges. Justify carefully.
\[ \sum_{n=1}^{\infty} \frac{(n!)^2}{(2n + 1)!} \]

11. Determine if the series converges or diverges. Justify carefully.
\[ \sum_{n=1}^{\infty} \tan\left(\frac{1}{n^2}\right) \]

12. Determine if the series converges absolutely, converges, or diverges. Justify carefully.
\[ \sum_{n=1}^{\infty} (-1)^n \frac{n^3}{3^n} \]

13. Determine if the series converges absolutely, converges, or diverges. Justify carefully.
\[ \sum_{n=1}^{\infty} (-1)^n \left(\sqrt[n]{n+1} - \sqrt[n]{n}\right) \]

14. Determine if the series converges absolutely, converges, or diverges. Justify carefully.
\[ \sum_{n=1}^{\infty} \frac{\cos\left(\frac{n\pi}{2}\right)}{n} \]
15. Determine if the series converges absolutely, converges, or diverges. Justify carefully.
\[ \sum_{n=1}^{\infty} \frac{\sin n}{n^2} \]

16. How many terms do you need to add to get a number that is within 0.0001 of the infinite sum for
\[ \sum_{n=0}^{\infty} \frac{(-1)^n}{(4n + 3)^3} \]

17. How many terms do you need to add to get a number that is within 0.01 of the infinite sum for
\[ \sum_{n=0}^{\infty} \frac{(-1)^n}{n^2 + 7} \]

18. How many terms do you need to add to get a number that is within 0.0001 of the infinite sum for
\[ \sum_{n=0}^{\infty} \frac{1}{3n + 1} \]

Explain your answer.

19. Find the interval of convergence and the radius of convergence for
\[ \sum_{n=0}^{\infty} \frac{\sqrt[n]{nx^n}}{5^n} \]

20. Find the interval of convergence and the radius of convergence for
\[ \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n} \]

21. Find the interval of convergence and the radius of convergence for
\[ \sum_{n=0}^{\infty} n^2 x^n \]

22. Find the interval of convergence and the radius of convergence for
\[ \sum_{n=0}^{\infty} \frac{(x - 2)^n}{n + 1} \]

23. Find the interval of convergence and the radius of convergence for
\[ \sum_{n=0}^{\infty} n! x^n \]

24. Find the interval of convergence and the radius of convergence for
\[ \sum_{n=0}^{\infty} \frac{2n + 1}{2^n + 3^n} (x - 1)^n \]

25. Compute the derivative of the power series and compare the intervals of convergence of the series and its derivative.
\[ \sum_{n=1}^{\infty} \frac{(x - 2)^n}{n} \]

26. Compute the derivative of the power series and compare the intervals of convergence of the series and its derivative.
\[ \sum_{n=0}^{\infty} \frac{x^n}{n!} \]

27. Find the power series for the function \( \sqrt[3]{(1 + x)^4} \) expanded about \( x = 0. \)
28. Find the power series for the function
\( \sqrt[6]{(1 + x)^6} \) expanded about \( x = 0 \).

29. Find the power series for the function
\( \left( \frac{1}{x+2} \right)^3 \) expanded about \( x = 0 \).

30. Find the third degree Maclaurin polynomial for the function \( f(x) = \tan x \).

31. Find the fourth degree Taylor polynomial for the function \( \ln x \) expanded about \( x = 1 \).

32. Devise the Taylor series for \( \sin x \) expanded about \( x = 0 \).

33. Derive the Maclaurin series for \( e^x \).

34. Use the error term for the Maclaurin series for \( e^x \) to estimate how close \( 1 + x + \frac{x^2}{2} \) is to \( e^x \) if \(-0.1 \leq x \leq 0.1\).

35. What degree Maclaurin polynomial does one need in order to approximate \( \cos x \) to within 0.00005 for \( 0 \leq x \leq 0.5 \)?

36. What degree Maclaurin polynomial does one need in order to approximate \( e^x \) to within 0.000005 for \(-1 \leq x \leq 1 \)?

37. Derive the power series for \( \tan x \) (expand about \( x = 0 \)). Use this to compute the sum of the series
\[ 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots \]

38. Derive the Maclaurin series for \( \ln(1 + x) \). Use this to compute the sum of the series
\[ 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots \]