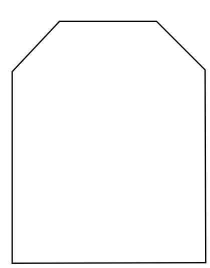
The Story. Several years ago the design of soda cans changed. The general shape did not change, but the dimensions changed. The top of the can was and still is shaped like a frustum of a cone. Today the top radius is smaller than it was in the old days and this change has been reported to save the industry approximately $1,000,000 per week in the cost of aluminum. The cost of producing a can depends on the amount of aluminum needed to make the can and other costs associated with the manufacturing process. In this project, you will redesign a soda can in order to minimize the amount of aluminum used per can as measured by the surface area.

The problem. Soda cans today have a diameter of 2.5 inches, a height of 4.8 inches and a volume of 21.7 cubic inches. In order to simplify the computations, you are to assume that soda cans consist of two parts, a cylindrical bottom part with a frustum of a cone on top. You are allowed to change the diameter and the heights of the cylindrical and frustum parts of the can as long as you keep the current volume and total height of the can at the current values. The top diameter of the frustum is also to remain at its current value of 2.125 inches. Your job is to design a can that will minimize surface area (total material used) with the given constraints and assumptions. Assuming the thickness of aluminum in most soda cans is 0.0047 inches, give an estimate of the amount of aluminum saved per can if your change were implemented and then estimate how much aluminum and money would be saved annually in the US manufacture of aluminum cans assuming that the industry produces approximately 24 billion soda cans per year, aluminum sells for $0.75 per pound and the density of aluminum is 167 pounds per cubic foot. Of course there are many issues that would have to addressed before the industry would change the can's design. For example, the redesigned can would have to work in the current vending machines and retooling costs would have to be considered. These issues are beyond the scope of this project.



Procedure. Although it is possible to do the calculations by hand, along the way the computations would be very tedious and in the end an equation would have to be solved which would be very difficult to solve without the aid of a calculator or computer. So you are strongly encouraged to use technology to help solve the problem. You can use Mathematica, Maple, Matlab, or other software products that do symbolic and numeric computations. Whatever product you use, you may wish to look at the help window to see how to do the following. (The Mathematica function for this is given in parentheses.)

1. Define a function. (For example, F[x_]:= x^2 is used to define the function $F(x) = x^2$)
2. Plot a graph (Plot)
3. Compute a derivative (If F is the function, F' is its derivative)
4. Numerically solve an equation (FindRoot)

Solve the problem like you would any max/min problem, except use a computer to assist you in the computations. When you find your solution, write your report stating what you did and any print outs of computer computations that you used to solve the problem. Since I am asking for print outs, it may not be feasible to use Wolfram Alpha. Your report should be written as if you were presenting your findings to your boss. That is, give an overview stating the dimensions you found to be optimal, explain what you did, and support any assertions you make with computations. In the end, be sure to state clearly what the radius of the can should be as well as what the height of the cylindrical and frustum parts of the can should be.