Please complete the SETE survey to evaluate the course before May 4. Why not do it now?

1. Use the definition to derive the formula for the derivative of $f(x) = \frac{x}{x+1}$.

2. Use the definition of derivative to derive $f'(x)$ where $f(x) = \sqrt{3x+1}$.

3. Use the definition of derivative to derive $g'(x)$ where $g(x) = \dfrac{2x+1}{x-1}$.

4. Derive the formula for the derivative of $\sin x$.

5. Use the derivative formulas for $\sin x$ and $\cos x$ to derive the formula for the derivative of $\tan x$.

6. Let $f(x) = x^2 + \dfrac{16}{x}$ for $1 \le x \le 4$. Find the maximum and minimum values for $f(x)$ on this interval. (Be sure to say how you know your answer is correct.)

7. Let $f(x) = x^3 - 4x^2 - 3x + 2$. Find the minimum and maximum values of $f(x)$ for $-2 \le x \le 2$. Be sure to justify all your conclusions.

8. An object moves along a straight line with position given by $s(t) = t^2 - 2t + 3$. Find the velocity and acceleration of the object at time $t = 2$.

9. An object moves along a straight line with position given by $s(t) = 5\cos(\frac{2\pi t}{3}) + 12\sin(\frac{2\pi t}{3})$. Find the velocity and acceleration of the object at time $t = 3$.

10. Find an equation of the tangent line to $y = \sec^2 x$ at the point where $x = \frac{\pi}{4}$.

11. Let $f(x) = \sqrt[3]{\frac{x}{2} + 1}$. Find the tangent line to $f(x)$ at the point where $x = 14$.

12. Find equations for all the lines that pass through the point $(0, -8)$ and are tangent to the graph of $y = 3x^2 + 4x + 9$. (Note that the point is not on the graph.)

13. Find all the horizontal and vertical asymptotes of the graph of $f(x) = \frac{3x^3+2x-9}{x^2-4}$.

14. Compute $\lim_{x\to 0} \frac{2\sin x + 3x}{x}$.

15. Compute $\lim_{x\to 0} \frac{\sec^2 x - 1}{3x \sin x}$.

16. Compute $\lim_{\theta\to 0} \frac{1-\cos^2\theta}{\theta^2}$.

17. Compute the derivative implicitly given that $\frac{4x^2y+2xy-2}{5x^2+3xy-y^3} = x$.

18. Find an equation of a line tangent to the curve defined by the equation $x^3y + xy + x^2y - 1 = 2xy^3$ at the point $(1, 1)$.

19. A cone made of very flexible material keeps its volume at $15\pi$ cubic feet while its radius and height vary. When the radius is 3 feet the radius is increasing at the rate of 0.1 feet per second. How fast is the height increasing?

20. Suppose that an object moves along a straight horizontal line with its displacement given by $s(t) = t^3 - 3t^2 + t - 1$. Find the velocity and acceleration of the object at all times $t$. Find all values of $t$ where the object is moving to the left.

21. On a certain planet the acceleration due to gravity is given by -5.7 m/sec$^2$. If a rock is thrown straight up at a velocity of 18 m/sec, how long will it take to hit the ground?

22. Let

$$f(x) = \begin{cases} \frac{\sin 2x}{x} & : \quad x < 0 \\ \frac{\tan 5x}{x} & : \quad 0 < x \\ 0 & : \quad x = 0 \end{cases}$$

Is $f$ continuous at $x = 0$? If so, show why. If not, determine if 0 is a removable discontinuity.

23. Let

$$f(x) = \begin{cases} 2x + 3 & : \quad x < 0 \\ 3x - 7 & : \quad 0 \le x \le 1 \\ 2x - 3 & : \quad x > 1 \end{cases}$$

Find

a) $\lim_{x \to 0^+} f(x)$

b) $\lim_{x \to 0^-} f(x)$

c) $\lim_{x \to 1^+} f(x)$

d) $\lim_{x \to 1^-} f(x)$

e) $\lim_{x \to 2^+} f(x)$

24. Use all the information you can concerning the first and second derivative and asymptotes to sketch the graph of

$$f(x) = \frac{x^3 + 32}{x^2}$$

25. Use all the information you can concerning the first and second derivative and asymptotes to sketch the graph of

$$g(x) = \frac{x^3}{3x^2 - 1}$$

26. Use all the information you can concerning the first and second derivative and asymptotes to sketch the graph of

$$f(x) = x^4 - 2x^2 + 4$$

27. Explain how the mean value theorem implies that if you average speed over some time interval is 70 miles per hour, then at some time in that interval you were traveling with a speed of 70 miles per hour.

28. Use the linearization of the appropriate function to estimate $\sin(\pi/3 + .01)$.

29. A beacon of light 200 m offshore rotates twice per minute, forming a spot of light that moves along a straight wall along the shore. How fast is the spot of light moving when the spot is at the point P, directly opposite the beacon? When it is 200 meters from point P?

30. A man walks directly under a street light. The man is 6 feet tall, the light is 15 feet above the ground, and the man walks at 3 feet per second. How fast is the shadow of the man growing when he is 5 feet from directly under the light?

31. A conical cup is to hold 100 cm$^3$. What dimensions will give it the least surface area?

32. A woman is in a boat 100 meters from shore. If she can row the boat at the rate of 1 meter per second and she can walk on shore at the rate of 2 meters per second, find her fastest path to a point 200 meters down the (straight) shore.

33. A rectangle is formed by making the lower left corner at the origin, one side along the $x$-axis, another side along the $y$-axis and the upper right corner on the graph of $y = \sqrt{9 - \frac{x^2}{4}}$. Find the point on the graph that maximizes the area of the rectangle.

34. Compute $\int \sin 2x\, dx$ in two different ways.

35. Compute $\int \frac{1}{x^2} \cos \frac{1-x}{x}\, dx$

36. Compute $\int_1^4 \frac{2x^2+3x-1}{\sqrt{x}}\, dx$

37. Compute $\int \frac{\sin^2 2x}{\cos^2 2x}\, dx$

38. Compute $\int_0^4 x\sqrt{x^2+9}\, dx$

39. Compute $\int x\sqrt{x^2+1}\, dx$

40. Compute $\int_1^4 \frac{x^2+2x+3}{\sqrt{x}}\, dx$

41. $\int_0^{\pi/2} \sin^2 x\, dx$

42. Compute $\int \frac{1}{\sqrt{x}\sqrt{\sqrt{x}+1)}}\, dx$

43. Compute $\int \csc^2\theta \cot^2\theta\, d\theta$

44. Use a limits of Riemann sums to compute $\int_1^4 (x^2+1)\, dx$.

45. Compute $\int_0^2 (x^2+x)\, dx$ using a limit of Riemann sums.

46. Compute $\lim_{n\to\infty} \Sigma_{k=1}^n \frac{(n+3k)^3}{n^4}$.

47. Compute $\lim_{n\to\infty} \Sigma_{k=1}^n \sin(\frac{k\pi}{n})\frac{1}{n}$.

    there is a constant $c$ such that $f(x) = g(x) + c$ for every real number $x$.

48. Find the area bounded by the functions $f(x) = \sin x$ and $g(x) = x(x-\pi)$.

49. Find the area bounded between the graphs of $x - y^2 + 1 = 0$ and $x - y - 1 = 0$.

50. Find the area bounded by the graphs of $y = 4x + 6$ and $y = x^2 + 3x + 4$.

51. Find the volume of a room whose floor is an ellipse with equation $\frac{x^2}{4} + \frac{y^2}{9} = 1$ and whose cross section is a semicircle when sliced perpendicular to the $x$-axis.

52. Derive the formula for the volume of a cone with base radius $r$ and height $h$.

53. The region $R$ is bounded between the functions $f(x) = x$ and $g(x) = \sqrt{x}$. Find the volume obtained when $R$ is rotated about the $x-$axis.

54. Find the volume generated if the region in the plane between $x = 0$ and $x = \pi$, below $y = \sqrt{\sin x}$, and above the $x$-axis is rotated about the $x$-axis.

55. A spherical tank with radius 2 is filled with water. The water is drained at the rate of 0.5 cubic meters per second. How fast is the water level in the tank changing at the instant that the water has a depth of 3 meters. (Depth of three meters means that the water level is one meter above the center.) I suggest that you first find the volume obtained from the region in the plane inside the circle $x^2+y^2=r^2$ and below the line $y=h$ for a fixed $h$ with $0\le h\le r$ rotated about the $y$-axis.

56. The region between the graphs of $y=x^2$ and $y=\frac{1}{8}+\frac{1}{2}x^2$ is revolved about the $y$-axis to form a lens. Compute the volume of the lens.

57. Find the volume of the solid formed by revolving the region enclosed by the graph of

$$x^{2/3}+y^{2/3}=a^{2/3}$$

about the $x$-axis.

58. Find the volume of the solid obtained by rotating the area in the plane bounded by $y=\sin(x^2)$, the $x-$axis, $x=0$ and $x=\sqrt{\pi/2}$ about the $y-$axis.

59. Find the center of mass of a wire that extend along the number line from 0 to 4 and whose cross section area when sliced perpendicular to the number line is a circle with radius $0.001x+0.1$.

60. Find the center of mass of the triangle with vertices $(0,0)$, $(3,0)$, and $(1,2)$.

61. Find the center of mass of the region in the first quadrant bounded by $y=x^2$ and $y=x^3$.

62. Find the center of mass of the region in the plane bounded by $y=x-3$ and $x=(y+3)(y-1)$.

63. Find the center of mass of the planar region bounded by $y=x^2$ and $y=x^3$. Now do the same thing with $y=x^n$ and $y=x^{n+1}$. Is the center of mass of a connected planar object always inside the object?

64. State the first form of the Fundamental Theorem of Calculus.

65. State the second form of the Fundamental Theorem of Calculus.

66. Find the area of the surface generated by revolving the graph of $y=x^3/3$ for $0\le x\le 1$ about the $x-$axis.

67. Find the area of the surface generated by revolving the graph of $x=\sqrt{y}$ for $2\le y\le 6$ about the $y-$axis.

68. Find the arc length of the graph of $y=\dfrac{5}{12}x^{6/5}-\dfrac{5}{8}x^{4/5}$ for $1\le x\le 32$.

69. The center of mass of a region in the upper half-plane is $(10,15)$. Find the volume of the solid created by revolving the area about the $x-$axis assuming that the region has area of 8.