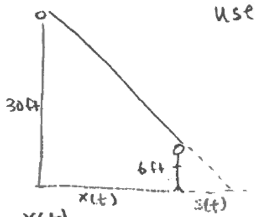
Related Rates and Max/Min

①



$x(t) = 10$ ft

$\frac{dx}{dt} = 8$ ft/s, $\frac{ds}{dt} = ?$

use similar triangles:

$$\frac{s(t)}{6\text{ ft}} = \frac{s(t) + x(t)}{30\text{ ft}}$$

$$s(t) + x(t) = 5\,s(t)$$

$$x(t) = 4\,s(t)$$

$$\frac{dx}{dt} = 4\frac{ds}{dt}$$

$$8 = 4\frac{ds}{dt}$$

$$2 = \frac{8}{4} = \frac{ds}{dt}$$

$\boxed{2 \text{ ft/s}}$

Ⅱ $f'(x) = 2(3x^2) - 5(2x) - 4(1) = 6x^2 - 10x - 4 = 2(3x+1)($[redacted]$x -$ [redacted]$2)$

critical values: $3x + 1 = 0$, $x = -\frac{1}{3}$ $\quad$ $x - 2 = 0$, $x = 2$

Interval $-1 \le x \le 3$: $\quad$ -1 $\;+\;$ $-\frac{1}{3}$ $\;-\;$ 2 $\;+\;$ 3

$x = -\frac{1}{2}$: $f'(-\frac{1}{2}) = (-\frac{1}{2})(-5) > 0$

$x = 0$: $f'(0) = (1)(-4) < 0$

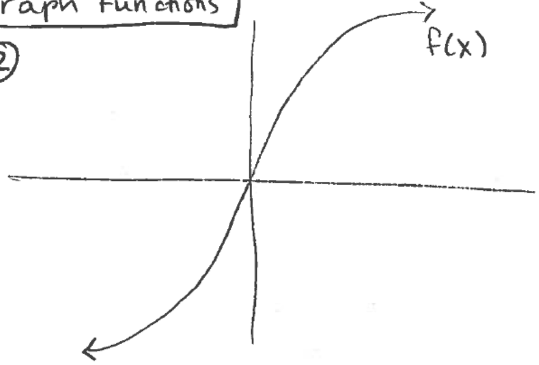
$x = \frac{5}{2}$: $f'(\frac{5}{2}) = (\frac{17}{2})(1) > 0$

$f(-\frac{1}{3}) = \frac{289}{27}$, $\quad f(2) = 2(2^3) - 5(2^2) - 4(2) + 10 = -2$

By First Derivative test, f has a local maximum at $(-\frac{1}{3}, \frac{289}{27})$ and a local minimum at (-2,2).

Using Derivatives to Graph Functions

②



$f'(x) > 0 \Rightarrow f(x)$ increasing; $f''(x) > 0 \Rightarrow$ cc up; $f''(x) < 0 \Rightarrow$ cc down

- No local minima/maxima on $(-\infty, \infty)$.
- Point of inflection at $(0,0)$.
- Function can have a horizontal asymptote, but does not necessarilly need one.