1. Explain how the mean value theorem implies that if you average speed over some time interval is 70 miles per hour, then at some time in that interval you were traveling with a speed of 70 miles per hour.

2. One car is traveling straight east from an intersection at 50 miles per hour. Another car is traveling straight north from the intersection at 60 miles per hour. How fast are the cars moving apart at the instant that the east bound car is two miles from the intersection and the north bound car is one mile from the intersection?

3. A 26 foot long ladder is leaning against a building. The bottom is pulled away from the building at the rate of 2 feet per second. How fast is the top dropping when the bottom of the ladder is 10 feet from the building?

4. Water is being pumped into a cone at the rate of 0.5 cubic feet per second. Suppose the cone is five feet tall and has a radius of six feet. (The vertex of the cone is on the bottom.) how fast is the level of water raising when the height of water is 2 feet?

5. A four foot tall child is running toward a building and directly away from a light on the ground. The child is running at four feet per second and she is exactly four feet from the light and sixteen feet from the wall. How fast is the height of her shadow on the wall shrinking?

6. Find a value of c that satisfies the Mean Value Theorem for the function $f(x) = x^2 + 4x + 3$, $a = 0$ and $b = 4$.

7. Find the value of c that satisfies the Mean Value Theorem for the function $f(x) = x^3$, $a = 0$ and $b = 4$.

8. Prove that if $f'(x) > 0$ for every x in the interval (a, b) and $f(x)$ is continuous over the interval $[a, b]$, then $f(x)$ is an increasing function on the interval $[a, b]$.

9. Prove that if a function has derivative 0 over an interval, then that function is a constant over that interval.

10. Prove that if $f'(x) = g'(x)$ for all x in an interval, then $f(x) = g(x) + c$, where c is some constant.

11. State conditions under which a continuous function must attain a maximum.

12. Give an example of a continuous function with domain $(0, 1)$ that does not have a maximum.

13. Give an example of a continuous function with domain $[0, \infty)$ that does not have a maximum.

14. Let $g(x) = 2x^3 + 3x^2 - 12x + 1$ for $-3 \leq x \leq 2$. Find the maximum and minimum of g in this domain.

15. Let $f(x) = x^{2/3}(x + 3)$ for $-1 \leq x \leq 2$. Find the maximum and minimum for f in this domain.

16. Let $f(x) = (x-1)\sqrt{x+2}$. Find all intervals where $f(x)$ is increase, decreasing, concave up and concave down.

17. Sketch the graph and find all local extrema, points of inflection, and asymptotes. Also indicate on which intervals the function is increasing, which intervals the function is decreasing, which intervals the function is concave up, and which intervals the function is concave down.

$$f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 12x + 8$$

18. Sketch the graph and find all local extrema, points of inflection, and asymptotes. Also indicate on which intervals the function is increasing, which intervals the function is decreasing, which intervals the function is concave up, and which intervals the function is concave down.

$$g(x) = x - 3x^{\frac{2}{3}}$$

19. Use all the information you can concerning the first and second derivative and asymptotes to sketch the graph of

$$f(x) = \frac{2x^2 - 2x - 4}{x^2 + x - 12}.$$

20. Sketch the graph of

$$f(x) = |\cos x| + 2.$$

21. A billboard 20 feet tall is located on top of a building with the billboard's lower edge 60 feet above the level of a viewer's eye. How far from a point directly below the sign should a viewer stand to maximize the angle between the lines of sight of the top and the bottom of the billboard?

22. A window is in the shape of a rectangle surmounted by a semicircle. If the perimeter of the window is 15 feet, find the dimensions that will allow the maximum amount of light to enter.

23. A fence 8 feet tall runs parallel to a building. The gap between the building and the fence is 1 foot. Find the length of the shortest ladder that will extend from the ground, over the fence and touch the building.

24. Find the point on the graph of $y = x^2 + 3$ that is closest to the point $(3, 3)$.

25. A wire 36 cm long is to be cut into two pieces. One piece is to be bent into the shape of a square, while the other is to be bent into the shape of a circle. Find the place to cut the wire in order to a) maximize the total area and b)minimize the total area.

26. Among all right circular cones with surface area 1, find the one with maximum volume.

27. Prove that the shortest line segment between a point (x_1, y_1) and the graph of a differentiable function $f(x)$ is a line segment that is perpendicular to the tangent line of the graph.

28. Sketch the graph using as much information as is feasible from the first and second derivatives and asymptotes.

$$y = x - 3x^{\frac{2}{3}}$$

29. Sketch the graph using as much information as is feasible from the first and second derivatives and asymptotes.

$$y = \frac{1}{8}(x^3 + 3x^2 - 9x - 27)$$

30. Find the limit $\lim_{x \to 0} \frac{\sqrt{\tan x}}{\sqrt{x}}$.

31. Find the limit $\lim_{x \to \infty} \left(\sqrt{x^2 + x} - \sqrt{x}\right)$.

32. Find the limit $\lim_{x \to 0} \frac{\cos x - 1 + x^2/2}{x^4}$

33. Find the limit

$$\lim_{x \to 1} \frac{x^6 + 5x^4 - 4x^2 + 2x - 4}{x^{10} - 5x^7 + 6x^4 + 3x^2 + x - 6}$$

34. Find the antiderivative

$$\int x^8 \, dx$$

35. Find the antiderivative

$$\int (x \cos x + \sin x) \, dx$$

36. Find the antiderivative

$$\int 3 \cos(2x + 1) \, dx$$

37. Find the antiderivative

$$\int \frac{7}{x^4} \, dx$$

38. Find the antiderivative of

$$\int 2 \sin(x) \cos(x) \, dx$$