1. Compute the limit and prove it is what you claim. $\lim_{x\to 3}(x^2+3x-1)$
2. Compute the limit and prove it is what you claim. $\lim_{x\to 10}(2x+1)$
3. Compute the limit and prove it is what you claim. $\lim_{x\to 5}(-x^2+6x+13)$
4. Compute the limit and prove it is what you claim. $\lim_{x\to 4}\frac{1}{x}$
5. Compute the limit and prove it is what you claim. $\lim_{x\to 6}\left(x+\frac{1}{x}\right)$
6. Compute the limit and prove it is what you claim. $\lim_{x\to 0}(x\sin x)$
7. Compute the limit and prove it is what you claim. $\lim_{x\to -3}\left(\frac{6x^2+2}{x-1}\right)$
8. Compute the limit and prove it is what you claim. $\lim_{x\to 1}((x-1)\cos x)$
9. Compute the limit and prove it is what you claim. $\lim_{x\to 53}(7x-9)$
10. Compute the limit and prove it is what you claim. $\lim_{t\to -2}(t^4+3t^3-2t+1)$
11. Compute $\lim_{x\to 2}\frac{3x^3+2x^2+3x-38}{x^2-3x+2}$. (No proof needed.)
12. Compute $\lim_{x\to 1}\frac{x^3+2x^2-4x+1}{2x^3-4x^2+x+1}$. (No proof needed.)
13. Compute $\lim_{t\to -1}\frac{t^3+2t^2-4t-5}{t^3+t^2+t+1}$. (No proof needed.)
14. Explain why $\lim_{x\to 2}\frac{2x}{x-2}$ does not exist.
15. Explain why $\lim_{x\to -1}\frac{x+1}{|x+1|}$ does not exist.
16. Explain why $\lim_{x\to 0}\sin\frac{1}{x}$ does not exist.
17. Explain why $\lim_{x\to 1}\frac{x^2+3x+1}{x^2-3x+2}$ does not exist.
18. Use the definition of derivative to compute $f'(x)$ given that $f(x)=3x^2+2x-9$
19. Use the definition of derivative to compute $f'(x)$ given that $f(x)=\frac{2x+3}{3x+2}$
20. Use the definition of derivative to compute $f'(x)$ given that $f(x)=\sqrt{x+2+3}$
21. Use the definition of derivative to compute $f'(x)$ given that $f(x)=\frac{1}{\sqrt[3]{x+1}}$
22. Use the definition of derivative to compute $f'(0)$ given that $f(x)=\cos x$
23. Use the definition of derivative to compute $f'(x)$ given that $f(x)=\frac{\sqrt{x^2+3x-1}}{2x+4}$
24. Find an equation of the tangent line to $y=3x^2+2x-5$ at the point $(1,0)$.
25. Find an equation of the tangent line to $f(x)=\frac{3x+2}{x-1}$ at the point $(3,11/2)$.

26. Find an equation of the tangent line to $g(x) = \sqrt{x+3}$ at the point $(6, 3)$.

27. Find an equation of the tangent line to $y = \sin(3x)$ at the point $(0, 0,)$.

28. Let

$$f(x) = \begin{cases} \frac{\sin 3x}{x} & : \quad x < 0 \\ \frac{\tan 2x}{x} & : \quad 0 < x \\ 0 & : \quad x = 0 \end{cases}$$

Is f continuous at $x = 0$? If so show why. If not, determine if 0 is a removable discontinuity.

29. Carefully state the definition of the $\lim_{x \to z} f(x) = l$ and then carefully state its negation.

30. Let

$$f(x) = \begin{cases} 2x + c & : \quad x < 1 \\ 5x - c & : \quad 1 \le x \end{cases}$$

Is there a value of c that makes f continuous at $x = 1$? If so, find c. If not, show why.

31. Prove: $\displaystyle\lim_{x \to 0} \frac{\sin x}{x} = 1$.

32. Prove: $\displaystyle\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$. (You may use Problem 31.)

33. Compute $\displaystyle\lim_{x \to 0} \frac{\sin x + x}{x}$.

34. Compute $\displaystyle\lim_{x \to 0} \frac{\tan x}{x}$.

35. Compute $\displaystyle\lim_{\theta \to 0} \frac{1 - \cos^2 \theta}{\theta^2}$.

36. Compute $\displaystyle\lim_{t \to 0} t \csc t$.

37. Compute $\displaystyle\lim_{t \to 0} \frac{\sin(2t)}{3t}$.

38. Prove $\displaystyle\lim_{x \to 3} \sqrt{x^2 + 7} = 4$

39. Prove $\displaystyle\lim_{x \to 2} \frac{5x^2 + 2x - 2}{x^2 - x - 4} = -11$

40. Compute $\displaystyle\lim_{x \to 3^+} f(x)$ and $\lim_{x \to 3^-} f(x)$. Does $\lim_{x \to 3} f(x)$ exist? If so state how you know, if not state why.

$$f(x) = \begin{cases} \sin x & : \quad x \le 0 \\ x & : \quad 0 < x \le 3 \\ x^2 - 4 & : \quad x > 3 \end{cases}$$

41. Compute $\lim_{x \to 0^+} f(x)$ and $\lim_{x \to 0^-} f(x)$. Does $\lim_{x \to 0} f(x)$ exist? If so state how you know, if not state why.

$$f(x) = \begin{cases} \sin x & : \quad x \le 0 \\ x & : \quad 0 < x \le 3 \\ x^2 - 4 & : \quad x > 3 \end{cases}$$

42. Is there a number c that makes the function f continuous, where f is defined below? If so, find c.

$$f(x) = \begin{cases} x^2 + 3 & : \quad x \le 1 \\ -2x^2 + c & : \quad x > 1 \end{cases}$$

43. Use induction to prove that $n^3 + n$ is divisible by 3 for any positive integer n.

44. Use induction to prove for any natural number n, $\displaystyle\sum_{k=0}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$.

45. Use induction to prove that for any natural number n, every polynomial of degree n has at most n roots. (You may use the fact that if $f(x)$ is a polynomial and $f(a) = 0$, then $x - a$ is a factor of $f(x)$.)

46. Let $x_0 = 3$, $x_1 = 1$ and for $n \geq 2$, $x_n = 2x_{n-1}+3x_{n-2}$. Prove that $x_n = 2(-1)^n + 3^n$ for every natural number n.

47. True or false- All cows have the same color.

48. Compute $\dfrac{d}{dx}(x^2 \sin(2x))$

49. Compute $\dfrac{d}{dx}\dfrac{x^2+1}{x^2+x-4}$

50. Compute $\dfrac{d}{dx}(\tan(x+3))^3$

51. Compute $\frac{d}{dx}\sqrt[5]{(x^3+2)^2}$

52. Compute $\dfrac{d}{dx}\dfrac{\sin x}{1+\sec x}$

53. Compute $\dfrac{d}{dx}\dfrac{\csc^2 x+3}{\pi}$

54. Use induction to prove the $\dfrac{d}{dx}x^n = nx^{n-1}$ for all positive integers n.

55. Find an equation of the tangent line to $y = \sin^3 x \cos x$ at the point where $x = \frac{\pi}{4}$.

56. Find an equation of the tangent line to $y = x \sin x$ at the point where $x = \frac{\pi}{3}$.

57. Find an equation of the tangent line to $y = \sqrt{3x-2}$ at the point where $x = 6$.

58. Find equations for all the lines that pass through the point $(2, 10)$ and are tangent to the graph of $y = x^2 + 2x + 3$. (Note that the point is not on the graph.)

59. Find equations for all the lines that pass through the point $(1, 50/9)$ and are tangent to the graph of $y = \frac{10}{x}$. (Note that the point is not on the graph.)

60. Prove the product formula for derivatives.

61. Prove the quotient formula for derivatives.

62. Use the linearization of the appropriate function to estimate $\sin(.1)$.

63. Use the linearization of the appropriate function to estimate $\sqrt[3]{999}$.

64. While driving across the country Tom is keeping track of his gas mileage. After getting back on the interstate highway he remembered that he did not compute his gas mileage. He remembers that he traveled 300 miles and used 14.8 gallons of gas. Explain how Tom can use a linear approximation to get a very good approximation of his gas mileage without taking his hands of the steering wheel. Do the approximation and compare your answer with the actual mileage.

65. Back in the ancient days before calculators (so long ago that your instructor

was young)every trigonometry book had a table of values for trigonometric functions. A certain table listed the values of $\cos x$ at .1 degree increments. When you look at the table, you notice that near 30 degrees the difference between successive entries in the table are approximately the same. What is that difference? (Use a linear approximation.)

66. Look at the first few derivatives of the function $f(x) = \frac{1}{x}$ and guess a formula for $f^{[n]}(x)$, the n^{th} derivative of $f(x)$. Show that the formula is correct for any natural number n.

67. Show that for any polynomial $f(x)$ of degree n, the n^{th} derivative of $f(x)$ is a constant given by the leading coefficient of $f(x)$ times $n!$.

68. Compute y' given that $x^4 + y^4 = a^4$ where a is a constant. Find equations of the lines tangent to this function at the points where $x = a/\sqrt[4]{2}$.

69. Compute $\frac{dy}{dx}$ given that $x^2y + y^2x - xy = 1$. Also find an equation of the tangent line through $(1, 1)$.

70. The position of an object along a line is given by $x(t) = 10t + \sin(3t)$ where t is measured in seconds and x is measured in meters. Find the object's velocity and acceleration at time t. State the units.

71. The velocity of an object is given by $v(t) = t^2 + 3t - 7$ where the distance units are feet and the time units are seconds. Find a formula for position and a formula for the acceleration and state the units for each.

72. The acceleration of an object is given by $a(t) = 28t$ where the distance unit is meters and the time unit is minutes. Find formulas for the velocity and position of the object. State the units of each.