## Group Project Rules and Encouragement

First, this is a major, lengthy assignment. You should start today and work on it some every day until you complete the project. Your paper will be due at the start of class on December 5, 2012.

1. If you have not already done so decide who will be in your group and plan to meet as a group today or tomorrow.
2. Start today. If you have the problem well in mind, your subconscious will work for you. You may find that good ideas will come to you at the strangest times.
3. Today, start by reading the entire project so you can see what the big picture is. Don't worry about details at first. Do this before the group meets so everyone has the overall picture in mind. You should probably discuss the project to be sure everyone has it well in mind and everyone agrees what the project is saying.
4. Next read the project in detail. Begin to think about the details you are to work out. At first it should go smoothly, but at some point you may hit a snag. At that point you will zero in on the obstacles. Keep thinking about the obstacles. You will have them well enough in mind that you will be able to work on the problems any time you have a spare minute or two. You may wish to keep a journal to record your progress every day.
5. You are to work as a group. You should discuss your progress as a group and help each other through the tough spots. You may discuss your progress on the project in general terms with friends outside your group, but do not give hints or help to other groups. Your group is not alone in this project. If you hit a snag and put substantial effort into the project with no progress, one person from your group should go to your teacher's office and ask for a hint. Only one person from the group is to go to your teacher's office at a time. That person will then be responsible to communicate to the others in the group what was said. One of the objectives of a group project is to give you practice in communicating mathematical ideas to others, so if your group seeks help more than once, take turns going to your teacher's office.
6. Do not go to anyone but your teacher for help on the project. The purpose is not to reproduce what someone else has already done, but to figure it out yourself. You may use references, but not to look up formulas or derivations. Use the referneces to learn more about spindles.
7. When you have completed the work on the project it is time to prepare it in written form. Your group will prepare only one report, not one for each member of the group. Your paper should be a mix of equations, graphs, formulas, and prose to support your conclusions. Pictures are also sometimes helpful. Use complete sentences, good grammar and correct punctuation. Spelling is also important. The prose is written to convey to the reader an explanation of what you have done. It should be written in such a way that an A student in another section of calculus would be able to understand it without the project directions. You will be graded on your written presentation as well as the mathematical content. Be sure each member of the group understands everything that was done on the project. You may have to meet a few times after the report is written to be sure everyone knows all the details.
8. After you have completed the project think about how much work each member of your group did on the project. Each person in the group is to turn in the Group Evaluation Form. Your response to the form will be seen only by you and your instructor. For some of the groups a grade may not be assigned until after an interview with your teacher. Your individual grade will usually be determined by the written report only. On rare occasions, when someone did considerably less than his or her share, an individual's grade will be determined by the report, how much of the project he or she did and in certain cases an interview. In most cases the letter grade for each member of the group is the same.

By signing below, you are stating that you have followed the project rules.

## Group Evaluation Form

## Name

1. List the names of the other members of your group. Next to each name, write a few words indicating how that person contributed and your best estimate of what percent of the project was done by this is person (include yourself). (For example, Sue - typed the report and solved part $2-25 \%$ )
2. Did some in your group show special insight on how to solve parts of the problem? If so, please indicate who and describe what they did.
3. Did the group work well together? Explain.
4. Do you prefer group projects or individual projects?

## Tibetan Spindles

Purpose. The first purpose of this project is to apply integration to a real world situation in order to design a useful product. The second purpose is to investigate how Riemann sums and their limits can be used to compute quantities that are useful in physics.

The story. Fleegle is an experienced spinner and she wishes to add a new and interesting spindle to her collection. Fleegle likes Tibetan spindles. Tibetan spindles are characterized as having a shaft to wind the yarn on and a whorl to keep the spindle spinning. She wants her new spindle to have similar spin characteristics to her favorite Tibetan spindles, but she would like the shape of the whorl to be different and she wants an atrtractive wood combination. Knowing that you are both an expert in calculus and very artistic, she asks for your help. Fleegle gives you the dimensions and the type of wood used for a couple of spindles that spin well and she wants the spindle that you design to spin as well as these. Your job is to design a spindle for Fleegle.

Procedure. You are to follow the outline below.

1. The moment of inertia $I$ is the main characteristic that determines how an object spins. For a point mass of $m$ that is a distance of $r$ from the axis of rotation, the moment of inertia is $m r^{2}$. For any finite number of point masses, the total moment of inertia is the sum of each. For a solid object, such as a top or spindle, one needs to take limits of Riemann sums to work out the integral to find the moment of inertia. For this part, find the formula for the moment of inertia of a solid with density $\rho$ formed by rotating the region in the plane bounded by the $x$-axis, $y=f(x), x=a$ and $x=b$ about the $y$-axis. (Assume only that $f(x) \geq 0$ and $f$ is continuous at $x$ for all $a \leq x \leq b$.)
2. Find the mass and moment of inertia of a cylinder with radius $r$, height $h$ and density $\rho$ whose axis of rotation is the axis of the cylinder. Show that for a cylinder, $I=\frac{M r^{2}}{2}$ where $M$ is the mass of the cylinder.
3. Find the moment of inertia of a cone with radius $r$, height $h$ and density $\rho$. Find a formula for the moment of inertia of a cone that involves only the mass and the radius of the cone (similar to the formula in part 2).
4. Next find the mass and the moment of inertia of a solid with density $\rho$ whose shape is the solid of revolution obtained by rotating the area in the first quadrant bounded by the parabola $y=-a x^{2}+h$ and the coordinate axes.
5. Use your results in part 4) to show that for a parabolic shaped whorl, $I=\frac{M r^{2}}{3}$ where $M$ is the mass and $r$ is the radius of the whorl.
6. Next find the mass and the moment of inertia of a spherical shaped whorl having radius $r$ and density $\rho$. (These spindles are called bead spindles.) Then use your formulas to get a formula for $I$ that involves only the mass and the radius (not the density) of the whorl.
7. Fleegle tells you that she has a spindle whose spin she really likes and it has a parabolic whorl made of snakewood with height 2.0 cm and radius 2.2 cm . She also has a nice bead spindle whose bead has radius 2 cm and mass 17 g . Find the moment of inertia
of each whorl. Based on the fact that these spindles spin well, approximately what moment of inertia do you think you should try for when designing a good spindle?
8. The shaft of a typical spindle is roughly a cone with radius no more than 0.5 cm , height no more than 25 cm and density no more than $1.4 \mathrm{~g} / \mathrm{cc}$. Based on this, give an estimate of the moment of inertia for a typical shaft. How do the moments of inertia of a shaft and the whorls in part 7 compare?
9. Suppose that you wanted to make a spindle with an ebony parabolic whorl. If the height of the whorl is to be 2.0 cm , what should the diameter be? Next suppose you wanted to make a spherical whorl out of holly, what radius would you use?
10. Now it is time to design your own spindle. You are to choose attractive woods for the whorl and the shaft as well as a nice shape for the whorl, keeping in mind that the whorl must have a good moment of inertia. The shape should be a solid of revolution. The function rotated may be defined piecewise, as one formula, or simply a drawing with distances marked. However you specify the function, you will need to show the computations for the moment of inertia. You should at least specify the wood to be used for the shaft and if you wish, you may specify the shaft's shape as well. The spindle should be between 22 and 28 cm long.

Helpful web site:
http://www.hobbithouseinc.com/personal/woodpics
http://www.engineeringtoolbox.com/wood-density-d_40.html
http://www.ravelry.com Look at the forums Spindle Candy, Support Spindlers, and Spindle Lore

Note: I plan to make your spindle over the break. I should have no trouble finding any of the following woods:

Walnut, Maple, Ebony, Cherry, Zebrawood, Tulipwood, Canarywood, Mesquite, Bois d'arc, Sycamore, Bocote, Macacauba, Teak, Yellowheart, Purpleheart, Redheart, Bloodwood, Holly, Ziricote, Limba (white or black), Kingwood, Oak burl (whorl only, not shaft), Jatoba (shaft only, not whorl), Ironwood (shaft only, not whorl), Mahogany, African Blackwood, and Goncalo Alves.

If you specify a different wood, I will try to get it, but I may have to make a substitution.
I plan to make one spindle for each group design. Your group may either keep the spindle or donate it to raise money for a UNT math scholarship. If your group decides to keep the spindle, then I may make a second one to sell to raise scholarship money.

