Group Project Rules and Encouragement

First, this is a major, lengthy assignment. You should start today and work on it some every day until you complete the project. Your paper will be due at the start of class on October 2, 2009.

1. If you have not already done so decide who will be in your group and plan to meet as a group today or tomorrow.
2. Start today. If you have the problem well in mind, your subconscious will work for you. You may find that good ideas will come to you at the strangest times.
3. Today, start by reading the entire project so you can see what the big picture is. Don't worry about details at first. Do this before the group meets so everyone has the overall picture in mind. You should probably discuss the project to be sure everyone has it well in mind and everyone agrees what the project is saying.
4. Next read the project in detail. Begin to think about the details you are to work out. At first it should go smoothly, but at some point you may hit a snag. At that point you will zero in on the obstacles. Keep thinking about the obstacles. You will have them well enough in mind that you will be able to work on the problems any time you have a spare minute or two. You may wish to keep a journal to record your progress every day.
5. You are to work as a group. You should discuss your progress as a group and help each other through the tough spots. You may discuss your progress on the project in general terms with friends outside your group, but do **not** give hints or help to other groups. Your group is not alone in this project. If you hit a snag and put substantial effort into the project with no progress, **one person** from your group should go to your teacher's office and ask for a hint. Even if things are going well it may not be a bad idea to discuss what you are doing with your teacher just to be sure you are on the right track. Only one person from the group is to go to your teacher's office at a time. That person will then be responsible to communicate to the others in the group what was said. One of the objectives of a group project is to give you practice in communicating mathematical ideas to others, so if your group seeks help more than once, take turns going to your teacher's office.
7. **Do not go to anyone but your teacher for help on the project. This includes other people, books, the web, or any other source of information. The purpose is not to reproduce what someone else has already done, but to figure it out yourself.**
8. When you have completed the work on the project it is time to prepare it in written form. Your group will prepare only one report, not one for each member of the group. Your paper should be a mix of equations, formulas, and prose to support your conclusions. Pictures are also sometimes helpful. Use complete sentences, good grammar and correct punctuation. Spelling is also important. The prose is written to convey to the reader an explanation of what you have done. It should be written in such a way that an A student in another section of calculus would be able to understand it without the project directions. You will be graded on your written presentation as well as the mathematical content. Be sure each member of the group understands everything that was done on the project. You may have to meet a few times after the report is written to be sure everyone knows all the details.
9. After you have completed the project think about how much work each member of your group did on the project. Each person in the group is to turn in the Group Evaluation Form. Your response to the form will be seen only by you and your instructor. For some of the groups a grade may not be assigned until after an interview with your teacher. Your individual grade will be determined by the written project, how much of the project you did, and in certain cases the interview. In most cases the letter grade for each member of the group should be the same.

By signing below, you are stating that you have followed the project rules.

Name   Date

Group Evaluation Form

Name

1. List the names of the other members of your group. Next to each name, write a few words indicating how that person contributed. (For example, Sue - typed the report and solved part 2.)

2. Did some in your group do substantially more than their share? If so, please indicate who and describe what they did.

3. Did some in your group do substantially less that their share? If so, please indicate who and describe what they did.

4. Did the group work well together? Explain.

5. Do you prefer group projects or individual projects?

Limits of Polynomials

Tom is taking Calculus 1710. Tom's sister, Mary, took calculus a few years ago and did quite well. Unfortunately for Tom she has forgotten some of the material on limits. Tom claims that if you want to compute the limit of a function then all you have to do is plug in the value for x into the function and you can compute the limit. Mary argues that that does not work for all functions. For example, $\lim_{x\to 0} \frac{x^2+x}{x} = 1$, but you can not plug in $x = 0$ to compute the limit.

Tom then says, "Well, yes. But, after you factor and cancel, then you can just plug in $x = 0$. I'll bet that for any function that you can plug in $x = a$, the limit $\lim_{x\to a} f(x)$ is the same as $f(a)$."

"What about the greatest integer function?" asks Mary. "$\lim_{x\to 0} \lfloor x \rfloor$ does not exist, but $\lfloor 0 \rfloor = 0$."

"You keep bringing in strange functions! I am talking about regular functions that you see every day. I still think that for any 'reasonable' function $\lim_{x\to a} f(x) = f(a)$."

"Oh, do you mean polynomial functions are continuous?" asks Mary.

"Yeah! Aren't they?"

"I really don't remember. But I do remember being surprised to hear my calculus teacher say that most functions are not continuous. So I think that some polynomial functions may not be continuous," Mary replied.

Tom and Mary go to you for help in determining the answer to this question. They realize that to show polynomials are continuous they only need to show that if $f(x)$ is a polynomial then $\lim_{x\to a} f(x) = f(a)$ for every real number a. Below is an outline of how you may prove that polynomials are continuous. You are not to use the properties of limits discussed in class such as $\lim_{x\to a}(f(x) + g(x)) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$, unless you prove them.

There is a fact involving inequalities which is quite useful in proving limits. First prove the following fact:

Fact: If a and b are real numbers then $|a + b| \leq |a| + |b|$. To prove this it may be useful to look at different cases, depending on whether a or b is positive or negative.

The goal of this project is to show that if f is a polynomial then $\lim_{x\to a} f(x) = f(a)$ for every real number a. (That is, f is continuous.) Follow the outline below and you should be successful.

(The numbers a and c are constants, while f and g are functions.)

1. Prove: $\lim_{x\to a} x = a$.
2. Prove: $\lim_{x\to a} c = c$ where c is a constant.
3. Prove: If $\lim_{x\to a} f(x) = L$, then $\lim_{x\to a} cf(x) = cL$ where c is a constant. (You may wish to do a special case where $c = 0$ and then do it for $c \neq 0$.)
4. Prove: If $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = K$, then $\lim_{x\to a}(f(x) + g(x)) = L + K$. (Hint: Given an ϵ find a δ so that when x is within δ of a, $f(x)$ is within $\frac{\epsilon}{2}$ of L and $g(x)$ is within $\frac{\epsilon}{2}$ of K.)
5. Prove: If $\lim_{x\to a} f(x) = L$, then $\lim_{x\to a} xf(x) = aL$.
6. Now use the above facts to prove by induction on the degree of the polynomial that if $f(x)$ is a polynomial, then $\lim_{x\to a} f(x) = f(a)$.