

1. Compute the limit and prove it is what you claim.  $\lim_{x \rightarrow 3} (x^2 + 3x - 1)$
2. Compute the limit and prove it is what you claim.  $\lim_{x \rightarrow 10} (2x + 1)$
3. Compute the limit and prove it is what you claim.  $\lim_{x \rightarrow 5} (-x^2 + 6x + 13)$
4. Compute the limit and prove it is what you claim.  $\lim_{x \rightarrow 4} \frac{1}{x}$
5. Compute the limit and prove it is what you claim.  $\lim_{x \rightarrow 6} \left(x + \frac{1}{x}\right)$
6. Compute the limit and prove it is what you claim.  $\lim_{x \rightarrow 0} (x \sin x)$
7. Compute the limit and prove it is what you claim.  $\lim_{x \rightarrow -3} \left(\frac{6x^2 + 2}{x - 1}\right)$
8. Compute the limit and prove it is what you claim.  $\lim_{x \rightarrow 1} ((x - 1) \cos x)$
9. Compute the limit and prove it is what you claim.  $\lim_{x \rightarrow 53} (7x - 9)$
10. Compute the limit and prove it is what you claim.  $\lim_{t \rightarrow -2} (t^4 + 3t^3 - 2t + 1)$
11. Compute  $\lim_{x \rightarrow 2} \frac{3x^3 + 2x^2 + 3x - 38}{x^2 - 3x + 2}$ . (No proof needed.)
12. Compute  $\lim_{x \rightarrow 1} \frac{x^3 + 2x^2 - 4x + 1}{2x^3 - 4x^2 + x + 1}$ . (No proof needed.)
13. Compute  $\lim_{t \rightarrow -1} \frac{t^3 + 2t^2 - 4t - 5}{t^3 + t^2 + t + 1}$ . (No proof needed.)
14. Explain why  $\lim_{x \rightarrow 2} \frac{2x}{x - 2}$  does not exist.
15. Explain why  $\lim_{x \rightarrow -1} \frac{x + 1}{|x + 1|}$  does not exist.
16. Explain why  $\lim_{x \rightarrow 0} \sin \frac{1}{x}$  does not exist.
17. Explain why  $\lim_{x \rightarrow 1} \frac{x^2 + 3x + 1}{x^2 - 3x + 2}$  does not exist.
18. Use the definition of derivative to compute  $f'(x)$  given that  $f(x) = 3x^2 + 2x - 9$
19. Use the definition of derivative to compute  $f'(x)$  given that  $f(x) = \frac{2x + 3}{3x + 2}$
20. Use the definition of derivative to compute  $f'(x)$  given that  $f(x) = \sqrt{x + 2} + 3$
21. Use the definition of derivative to compute  $f'(x)$  given that  $f(x) = \frac{1}{\sqrt[3]{x + 1}}$
22. Use the definition of derivative to compute  $f'(0)$  given that  $f(x) = \cos x$
23. Use the definition of derivative to compute  $f'(x)$  given that  $f(x) = \frac{\sqrt{x^2 + 3x - 1}}{2x + 4}$
24. Find an equation of the tangent line to  $y = 3x^2 + 2x - 5$  at the point  $(1, 0)$ .
25. Find an equation of the tangent line to  $f(x) = \frac{3x + 2}{x - 1}$  at the point  $(3, 11/2)$ .

26. Find an equation of the tangent line to  $g(x) = \sqrt{x+3}$  at the point  $(6, 3)$ .

27. Find an equation of the tangent line to  $y = \sin(3x)$  at the point  $(0, 0)$ .

28. Let

$$f(x) = \begin{cases} \frac{\sin 3x}{x} & : x < 0 \\ \frac{\tan 2x}{x} & : 0 < x \\ 0 & : x = 0 \end{cases}$$

Is  $f$  continuous at  $x = 0$ ? If so show why. If not, determine if 0 is a removable discontinuity.

29. Carefully state the definition of the  $\lim_{x \rightarrow z} f(x) = l$  and then carefully state its negation.

30. Let

$$f(x) = \begin{cases} 2x + c & : x < 1 \\ 5x - c & : 1 \leq x \end{cases}$$

Is there a value of  $c$  that makes  $f$  continuous at  $x = 1$ ? If so, find  $c$ . If not, show why.

31. Prove:  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$ .

32. Prove:  $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$ . (You may use Problem 12.)

33. Compute  $\lim_{x \rightarrow 0} \frac{\sin x + x}{x}$ .

34. Compute  $\lim_{x \rightarrow 0} \frac{\tan x}{x}$ .

35. Compute  $\lim_{\theta \rightarrow 0} \frac{1 - \cos^2 \theta}{\theta^2}$ .

36. Compute  $\lim_{t \rightarrow 0} t \csc t$ .

37. Compute  $\lim_{t \rightarrow 0} \frac{\sin(2t)}{3t}$ .

38. Prove  $\lim_{x \rightarrow 3} \sqrt{x^2 + 7} = 4$

39. Prove  $\lim_{x \rightarrow 2} \frac{5x^2 + 2x - 2}{x^2 - x - 4} = -11$

40. Compute  $\lim_{x \rightarrow 3^+} f(x)$  and  $\lim_{x \rightarrow 3^-} f(x)$ . Does  $\lim_{x \rightarrow 3} f(x)$  exist? If so state how you know, if not state why.

$$f(x) = \begin{cases} \sin x & : x \leq 0 \\ x & : 0 < x \leq 3 \\ x^2 - 4 & : x > 3 \end{cases}$$

41. Compute  $\lim_{x \rightarrow 0^+} f(x)$  and  $\lim_{x \rightarrow 0^-} f(x)$ . Does  $\lim_{x \rightarrow 0} f(x)$  exist? If so state how you know, if not state why.

$$f(x) = \begin{cases} \sin x & : x \leq 0 \\ x & : 0 < x \leq 3 \\ x^2 - 4 & : x > 3 \end{cases}$$

42. Is there a number  $c$  that makes the function  $f$  continuous, where  $f$  is defined below? If so, find  $c$ .

$$f(x) = \begin{cases} x^2 + 3 & : x \leq 1 \\ -2x^2 + c & : x > 1 \end{cases}$$

43. Use induction to prove that  $n^3 + n$  is divisible by 3 for any positive integer  $n$ .

44. Use induction to prove for any natural number  $n$ ,  $\sum_{k=0}^n k^2 = \frac{n(n+1)(2n+1)}{6}$ .

45. Use induction to prove that for any natural number  $n$ , every polynomial of degree  $n$  has at most  $n$  roots. (You may use the fact that if  $f(x)$  is a polynomial and  $f(a) = 0$ , then  $x - a$  is a factor of  $f(x)$ .)
46. Let  $x_0 = 3$ ,  $x_1 = 1$  and for  $n \geq 2$ ,  $x_n = 2x_{n-1} + 3x_{n-2}$ . Prove that  $x_n = 2(-1)^n + 3^n$  for every natural number  $n$ .
47. True or false- All cows have the same color.