Prove the following statements **using the method of mathematical induction.**

1. Guess the formula for $\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+\cdots+\frac{1}{n(n+1)}$ by looking at small values of $n$ and then prove your answer is correct for any positive value of $n$ using induction.

2. For every positive integer $n$, $\sum_{k=1}^{n}\frac{1}{2^k}=1-\frac{1}{2^n}$.

3. $1\times 1!+2\times 2!+\cdots n\times n!=(n+1)!-1$ for any natural number $n$.

4. $\sum_{k=0}^{n}k^3=\left(\frac{n(n+1)}{2}\right)^2$ for any positive integer $n$.

5. For any natural number $n$, $\frac{1}{1\times 3}+\frac{1}{3\times 5}+\cdots+\frac{1}{(2n-1)(2n+1)}=\frac{n}{2n+1}$.

6. For any natural number $n$, $2+9+16+\cdots+(7n-5)=\frac{n(7n-3)}{2}$.

7. The number $n^3-n$ is divisible by three for any positive integer $n$.

8. The number $n^5+4n$ is divisible by 5 for any natural number $n$.

9. The sum of the angles of any polygon with $n$ sides is $\pi(n-2)$.

10. Any postage of over 7 cents can be paid using only 3 cent stamps and 5 cent stamps.

11. For every positive integer $n$, $n^2\geq n$.

12. For any natural number $n$, $3n\leq 3^n$.

13. For any real number $x\geq -1$ and any natural number $n$, $(1+x)^n\geq 1+nx$.

14. For any natural number $n$ and any real number $x$, $|\sin nx|\leq n|\sin x|$.

15. For any natural number $n$, $\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\cdots+\frac{1}{\sqrt{n}}<2\sqrt{n}$.

16. Define the Fibinacci sequence by $f_1=1$, $f_2=2$, and for $n\geq 3$, $f_n=f_{n-1}+f_{n-2}$. Prove for any natural number $n$, $f_1+f_2+f_3+\cdots+f_n=f_{n+2}-1$.

17. Using the Fibinacci sequence defined in the previous problem, prove that $\sum_{k=1}^{n}f_k^2=f_nf_{n+1}$ for any natural number $n$.

18. Using the notation in the previous two problems show that $f_n=\frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n-\left(\frac{1-\sqrt{5}}{2}\right)^n\right]$ for any natural number $n$.