1. Use the $\epsilon - \delta$ definition of limit to prove $\lim_{x\to 3}(x^2 - 2x + 1) = 4$.

2. Use the $\epsilon - \delta$ definition of limit to prove $\lim_{x\to 2} \dfrac{x+7}{x+1} = 3$.

3. Prove that if $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = K$, then $\lim_{x\to a}(f(x) + g(x)) = L + K$.

4. Use the definition of derivative to derive $f'(x)$ where $f(x) = \sqrt{3x+1}$.

5. Use the definition of derivative to derive $g'(x)$ where $g(x) = \dfrac{2x+1}{x-1}$.

6. Derive the formula for the derivative of $\sin x$.

7. Use the derivative formulas for $\sin x$ and $\cos x$ to derive the formula for the derivative of $\tan x$.

8. Let $f(x) = x^2 + \dfrac{16}{x}$ for $1 \leq x \leq 4$. Find the maximum and minimum values for $f(x)$ on this interval. (Be sure to say how you know your answer is correct.)

9. An object moves along a straight line with position given by $s(t) = t^2 - 2t + 3$. Find the velocity and acceleration of the object at time $t = 2$.

10. An object moves along a straight line with position given by $s(t) = 5\cos(\frac{2\pi t}{3}) + 12\sin(\frac{2\pi t}{3})$. Find the velocity and acceleration of the object at time $t = 3$.

11. Find an equation of the tangent line to $y = \sec^2 x$ at the point where $x = \frac{\pi}{4}$.

12. Find equations for all the lines that pass through the point $(0, -8)$ and are tangent to the graph of $y = 3x^2 + 4x + -9$. (Note that the point is not on the graph.)

13. Find all the horizontal and vertical asymptotes of the graph of $f(x) = \frac{3x^3+2x-9}{x^2-4}$.

14. Compute $\lim_{x\to 0} \frac{2\sin x + 3x}{x}$.

15. Compute $\lim_{x\to 0} \frac{\sec^2 x - 1}{3x \sin x}$.

16. Compute $\lim_{\theta\to 0} \frac{1-\cos^2\theta}{\theta^2}$.

17. Compute the derivative implicitly given that $\frac{4x^2y+2xy-2}{5x^2+3xy-y^3} = x$.

18. Find an equation of a line tangent to the curve defined by the equation $x^3y + xy + x^2y - 1 = 2xy^3$ at the point $(1, 1)$.

19. A cone made of very flexible material keeps its volume at 15π cubic feet while its radius and height vary. When the radius is 3 feet the radius is increasing at the rate of 0.1 feet per second. How fast is the height increasing?

20. Suppose that an object moves along a straight horizontal line with its displacement given by $s(t) = t^3 - 3t^2 + t - 1$. Find the velocity and acceleration of the object at all times t. Find all values of t where the object is moving to the left.

21. On a certain planet the acceleration due to gravity is given by -5.7 m/sec^2. If a rock is thrown straight up at a velocity of 18 m/sec, how long will it take to hit the ground?

22. Let

$$f(x) = \begin{cases} \frac{\sin 2x}{x} & : & x < 0 \\ \frac{\tan 5x}{x} & : & 0 < x \\ 0 & : & x = 0 \end{cases}$$

Is f continuous at $x = 0$? If so, show why. If not, determine if 0 is a removable discontinuity.

23. Let

$$f(x) = \begin{cases} 2x + 3 & : & x < 0 \\ 3x - 7 & : & 0 \le x \le 1 \\ 2x - 3 & : & x > 1 \end{cases}$$

Find

a) $\lim_{x \to 0^+} f(x)$

b) $\lim_{x \to 0^-} f(x)$

c) $\lim_{x \to 1^+} f(x)$

d) $\lim_{x \to 0^-} f(x)$

e) $\lim_{x \to 2^+} f(x)$

24. Use all the information you can concerning the first and second derivative and asymptotes to sketch the graph of

$$f(x) = \frac{x^3 + 32}{x^2}$$

25. Use all the information you can concerning the first and second derivative and asymptotes to sketch the graph of

$$g(x) = \frac{x^3}{3x^2 - 1}$$

26. Use all the information you can concerning the first and second derivative and asymptotes to sketch the graph of

$$f(x) = x^4 - 2x^2 + 4$$

27. Explain how the mean value theorem implies that if you average speed over some time interval is 70 miles per hour, then at some time in that interval you were traveling with a speed of 70 miles per hour.

28. Given that $x_1 = 0$, find x_2 when using Newton's method to solve $x^3+4x^2-2x+3 = 0$.

29. Use the linearization of the appropriate function to estimate $\sin(\pi/3 + .01)$.

30. A beacon of light 200 m offshore rotates twice per minute, forming a spot of light that moves along a straight wall along the shore. How fast is the spot of light moving when the spot is at the point P, directly opposite the beacon? When it is 200 meters from point P?

31. A man walks directly under a street light. The man is 6 feet tall, the light is 15 feet above the ground, and the man walks at 3 feet per second. How fast is the shadow of the man growing when he is 5 feet from directly under the light?

32. A conical cup is to hold 100 cm^3. What dimensions will give it the least surface area?

33. A woman is in a boat 100 meters from shore. If she can row the boat at the rate of 1 meter per second and she can walk on shore at the rate of 2 meters per second, find her fastest path to a point 200 meters down the (straight) shore.

34. Compute $\int \sin 2x\,dx$ in two different ways.

35. Compute $\int \frac{1}{x^2}\cos\frac{1-x}{x}\,dx$

36. Compute $\int_1^4 \frac{2x^2+3x-1}{\sqrt{x}}\,dx$

37. Compute $\int \frac{\sin^2 2x}{\cos^2 2x}\,dx$

38. Compute $\int_0^4 x\sqrt{x^2+9}\,dx$

39. Compute $\int \frac{1}{\sqrt{x}\sqrt{\sqrt{x}+1)}}\,dx$

40. Compute $\int \csc^2\theta\cot^2\theta\,d\theta$

41. Use a limits of Riemann sums to compute $\int_1^4 (x^2+1)\,dx$.

42. Compute $\lim_{n\to\infty}\Sigma_{k=1}^{n}\frac{(n+3k)^3}{n^4}$.

43. Compute $\lim_{n\to\infty}\Sigma_{k=1}^{n}\sin(\frac{k\pi}{n})\frac{1}{n}$.

44. Prove the Mean Value Theorem using Rolles' Theorem.

45. Prove Rolles' Theorem.

46. Prove that if $f'(x)=g'(x)$ for every real number x, then there is a constant c such that $f(x)=g(x)+c$ for every real number x.

47. Find the area bounded by the functions $f(x)=\sin x$ and $g(x)=x(x-\pi)$.

48. Find the area bounded between the graphs of $x-y^2+1=0$ and $x-y-1=0$.

49. Find the volume of a room whose floor is an ellipse with equation $\frac{x^2}{4}+\frac{y^2}{9}=1$ and whose cross section is a semicircle when sliced perpendicular to the x-axis.

50. Find the value of n needed to estimate
$$\int_0^\pi \sin^3 x\,dx$$
using the trapezoid rule if you want your estimate to be within 0.0001 of the correct answer. Use the n you found to estimate the integral.

51. Find the value of n needed to estimate
$$\int_0^\pi \sin^3 x\,dx$$
using the Simpson's rule if you want your estimate to be within 0.0001 of the correct answer. Use the n you found to estimate the integral.

52. Derive the formula for the volume of a cone with base radius r and height h.

53. Find the volume generated if the region in the plane between $x=0$ and $x=\pi$, below $y=\sqrt{\sin x}$, and above the x-axis is rotated about the x-axis.

54. The region between the graphs of $y = x^2$ and $y = \frac{1}{8} + \frac{1}{2}x^2$ is revolved about the y-axis to form a lens. Compute the volume of the lens.

55. Find the volume of the solid formed by revolving the region enclosed by the graph of

$$x^{2/3} + y^{2/3} = a^{2/3}$$

about the x-axis.

56. Find the volume of the solid obtained by rotating the area in the plane bounded by $y = \sin(x^2)$, the $x-$axis, $x = 0$ and $x = \sqrt{\pi/2}$ about the $y-$axis.

57. Find the center of mass of a wire that extend along the number line from 0 to 4 and whose cross section area when sliced perpendicular to the number line is a circle with radius $0.001x + 0.1$.

58. Find the center of mass of the triangle with vertices $(0, 0)$, $(3, 0)$, and $(1, 2)$.

59. Find the center of mass of the region in the plane bounded by $y = x - 3$ and $x = (y + 3)(y - 1)$.

60. Find the center of mass of the planar region bounded by $y = x^2$ and $y = x^3$. Now do the same thing with $y = x^n$ and $y = x^{n+1}$. Is the center of mass of a connected planar object always inside the object?

61. State the first form of the Fundamental Theorem of Calculus. Prove it.

62. State the second form of the Fundamental Theorem of Calculus. Prove it. (You may use the first form in your proof.)