1. Sketch the graph and find all local extrema, points of inflection, and asymptotes. Also indicate on which intervals the function is increasing, which intervals the function is decreasing, which intervals the function is concave up, and which intervals the function is concave down.
$$f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 12x + 8$$
2. Sketch the graph and find all local extrema, points of inflection, and asymptotes. Also indicate on which intervals the function is increasing, which intervals the function is decreasing, which intervals the function is concave up, and which intervals the function is concave down.
$$g(x) = x - 3x^{\frac{2}{3}}$$
3. Use all the information you can concerning the first and second derivative and asymptotes to sketch the graph of
$$f(x) = \frac{2x^2 - 2x - 4}{x^2 + x - 12}.$$
4. Sketch the graph of
$$f(x) = |\cos x| + 2.$$
5. Explain how the mean value theorem implies that if you average speed over some time interval is 70 miles per hour, then at some time in that interval you were traveling with a speed of 70 miles per hour.
6. Use the linearization of the appropriate function to estimate $\sin(.1)$.
7. Use the linearization of the appropriate function to estimate $\sqrt[3]{999}$.
8. While driving across the country Tom is keeping track of his gas mileage. After getting back on the interstate highway he remembered that he did not compute his gas mileage. He remembers that he traveled 300 miles and used 14.8 gallons of gas. Explain how Tom can use a linear approximation to get a very good approximation of his gas mileage without taking his hands of the steering wheel. Do the approximation and compare your answer with the actual mileage.
9. Back in the ancient days before calculators (so long ago that your instructor was young)every trigonometry book had a table of values for trigonometric functions. A certain table listed the values of $\cos x$ at .1 degree increments. When you look at the table, you notice that near 30 degrees the difference between successive entries in the table are approximately the same. What is that difference? (Use a linear approximation.)
10. Sketch the graph of $f(x) = x^3 + 3x^2 - 9x + 3$, using the first and second derivatives. Then find all roots of the function numerically using Newton's method. (Give your answer to 5 decimal points.)
11. Let $f(x) = x^3 + 7x^2 - 2x + 4$. Find and

simplify the formula for x_{n+1} using Newton's method. Use $x_1 = 1$ and find x_2.

12. Look at the first few derivatives of the function $f(x) = \frac{1}{x}$ and guess a formula for $f^{[n]}(x)$, the n^{th} derivative of $f(x)$. Show that the formula is correct for any natural number n.

13. Show that for any polynomial $f(x)$ of degree n, the n^{th} derivative of $f(x)$ is a constant given by the leading coefficient of $f(x)$ times $n!$.

14. Use the mean value theorem to prove that any polynomial of degree n has at most n roots for any natural number n. (Hint: Let $S(n)$ be the statement that any polynomial of degree n has at most n roots. In the induction step think about the question "If $g(a) = 0$ and $g(b) = 0$, what does the Mean Value Theorem say about the derivative of g(x)?")

15. One car is traveling straight east from an intersection at 50 miles per hour. Another car is traveling straight north from the intersection at 60 miles per hour. How fast are the cars moving apart at the instant that the east bound car is two miles from the intersection and the north bound car is one mile from the intersection?

16. A 26 foot long ladder is leaning against a building. The bottom is pulled away form the building at the rate of 2 feet per second. How fast is the top dropping when the bottom of the ladder is 10 feet from the building?

17. Water is being pumped into a cone at the rate of 0.5 cubic feet per second. Suppose the cone is five feet tall and has a radius of six feet. (The vertex of the cone is on the bottom.) how fast is the level of water raising when the height of water is 2 feet?

18. A four foot tall child is running toward a building and directly away from a light on the ground. The child is running at four feet per second and she is exactly four feet from the light and sixteen feet from the wall. How fast is the height of her shadow on the wall shrinking?

19. A billboard 20 feet tall is located on top of a building with the billboard's lower edge 60 feet above the level of a viewer's eye. How far from a point directly below the sign should a viewer stand to maximize the angle between the lines of sight of the top and the bottom of the billboard?

20. A window is in the shape of a rectangle surmounted by a semicircle. If the perimeter of the window is 15 feet, find the dimensions that will allow the maximum amount of light to enter.

21. A fence 8 feet tall runs parallel to a building. The gap between the building and the fence is 1 foot. Find the length of the shortest ladder that will extend form the ground, over the fence and touch the building.

22. Find the point on the graph of $y = x^2 + 3$ that is closest to the point $(3, 3)$.

23. A wire 36 cm long is to be cut into two pieces. One piece is to be bent into the shape of a square, while the other is to be bent into the shape of a circle. Find the place to cut the wire in order to a) maximize the total area and b)minimize the total area.

24. Among all right circular cones with surface area 1, find the one with maximum volume.

25. Prove that the shortest line segment between a point (x_1, y_1) and the graph of a differentiable function $f(x)$ is a line segment that is perpendicular to the tangent line of the graph.

26. Sketch the graph using as much information as is feasible from the first and second derivatives and asymptotes.
$$y = x - 3x^{\frac{2}{3}}$$

27. Sketch the graph using as much information as is feasible from the first and second derivatives and asymptotes.
$$y = \frac{1}{8}(x^3 + 3x^2 - 9x - 27)$$

28. Prove the Mean Value Theorem using Rolles' Theorem.

29. Prove Rolles' Theorem.

30. Prove that if $f'(x) = 0$ on some interval, then $f(x)$ is a constant on that interval.

31. Prove that if $f'(x) = g'(x)$ for every x in an interval, then $f(x) = g(x) + C$ for some constant C and every x in the interval.

32. Compute y' given that $x^4 + y^4 = a^4$ where a is a constant. Find equations of the lines tangent to this function at the points where $x = a/\sqrt[4]{2}$.

33. Compute $\frac{dy}{dx}$ given that $x^2y + y^2x - xy = 1$. Also find an equation of the tangent line through $(1, 1)$.

34. The position of an object along a line is given by $x(t) = 10t + \sin(3t)$ where t is measured in seconds and x is measured in meters. Find the object's velocity and acceleration at time t. State the units.

35. The velocity of an object is given by $v(t) = t^2 + 3t - 7$ where the distance units are feet and the time units are seconds. Find a formula for position and a formula for the acceleration and state the units for each.

36. The acceleration of an object is given by $a(t) = 28t$ where the distance unit is meters and the time unit is minutes. Find formulas for the velocity and position of the object. State the units of each.

37. State conditions under which a continuous function must attain a maximum.

38. Give an example of a continuous function with domain $(0, 1)$ that does not have a maximum.

39. Give an example of a continuous function with domain $[0, \infty)$ that does not have a maximum.

40. A function has domain $[a, b]$. Prove that if a maximum for the function at $x = c$, then either c is an end point or c is a critical point.

41. Find the antiderivative of $f(x) = 5x^2 - 3x + 7$.

42. Find the antideriviate of $f(x) = 3\tan(2x)\sec(2x)$.

43. Solve the intial value problem $f'(x) = 3x^2 - 2x + 1$ and $f(1) = 2$.

44. Solve the initial value problem $f''(x) = \sin x + \cos x$, $f(0) = 1$ and $f'(0) = 0$.