1. A billboard 20 feet tall is located on top of a building with the billboard's lower edge 60 feet above the level of a viewer's eye. How far from a point directly below the sign should a viewer stand to maximize the angle between the lines of sight of the top and the bottom of the billboard?

2. A window is in the shape of a rectangle surmounted by a semicircle. If the perimeter of the window is 15 feet, find the dimensions that will allow the maximum amount of light to enter.

3. A fence 8 feet tall runs parallel to a building. The gap between the building and the fence is 1 foot. Find the length of the shortest ladder that will extend form the ground, over the fence and touch the building.

4. Find the point on the graph of $y = x^2 + 3$ that is closest to the point $(3, 3)$.

5. A wire 36 cm long is to be cut into two pieces. One piece is to be bent into the shape of a square, while the other is to be bent into the shape of a circle. Find the place to cut the wire in order to a) maximize the total area and b)minimize the total area.

6. Among all right circular cones with surface area 1, find the one with maximum volume.

7. Prove that the shortest path between a point (x_1, y_1) and the graph of a differentiable function $f(x)$ is a line segment that is perpendicular to the tangent line of the graph.

8. If your answer to this question is the number a, then the number of points you receive on this question is $\frac{0.1}{3.0715-f(a)}$, where $f(x) = \lceil \frac{x^2x-3}{x^2+1} \rceil$.

9. Compute $\displaystyle\int_0^1 \cos(\pi x)dx$.

10. Compute $\displaystyle\int_1^2 \frac{3x}{\sqrt[4]{2x^2+7}}dx$.

11. Compute $\displaystyle\int_4^{16} \frac{1-\sqrt{u}}{\sqrt{u}}du$ two different ways.

12. Compute $\displaystyle\int_0^1 t\sqrt[3]{t^2+2}dt$.

13. Compute $\displaystyle\int \frac{1}{\sqrt{x}\sqrt{\sqrt{x}+1}}\,dx$.

14. Compute $\displaystyle D_x \int_x^{x^2} 3t \cos\sqrt{t}dt$.

15. Compute $\displaystyle\frac{d}{dt}\int_{-t}^{2t} x^2 \frac{x-3}{x^2+1}dx$.

16. Let $\displaystyle F(x) = \int_0^{\tan x} \frac{1}{1+t^2}\,dt$ for $0 \le x < \frac{\pi}{2}$.

    a) Compute $F'(x)$ and simplify your answer as far as possible.

    b) Based on your answer to part a), find a simple formula for $F(x)$.

    c) Based on part b), find a formula for $\displaystyle\int \frac{1}{1+t^2}\,dt$.

d) Convert the formula in part c) to a formula involving a derivative instead of an integral.

17. Let $F(x) = \int_0^{\sin x} \frac{1}{\sqrt{1-t^2}}\,dt$ for $0 \le x \le \frac{\pi}{2}$.

a) Compute $F'(x)$ and simplify your answer as far as possible.

b) Based on your answer to part a), find a simple formula for $F(x)$.

c) Based on part b), find a formula for $\int \frac{1}{1+t^2}\,dt$.

d) Convert the formula in part c) to a formula involving a derivative instead of an integral.

18. Let $F(x) = \int_1^x \frac{1}{t}\,dt$ for any positive real number x. Show that

a) $F(1/a) = -F(a)$ for any positive real number a.

b) $F(ab) = F(a)+F(b)$ for all positive real numbers a and b.

c) $F(a^n) = nF(a)$ for all positive integers n.

d) $F(a^n) = nF(a^n)$ for all integers.

e) Based on parts a)-d), what function does $F(x)$ seem to be?

19. Find the area bounded between the x-axis and the function $f(x) = x^3 - 6x^2 + 8x$.

20. Find the area bounded between the function $f(x) = 2(x^3 - x)$ and the function $f(x) = x^3$.

21. Find the area bounded between the graphs of $y = 2\sin x$ and $y = \sin(2x)$ where $0 \le x \le \pi$.

22. Write Simpson's rule for $\int_0^{\pi} \sin x^2 dx$ using $n = 6$. Find an n that according to the error estimate on Simpson's rule gives you an error of at most 0.000005 for this integral.

23. Write the trapezoid rule for $\int_0^{\pi} \sin x^2 dx$ using $n = 6$. Find an n that according to the error estimate on the trapezoid rule gives you an error of at most 0.000005 for this integral.

24. How large does n have to be in order to compute $\int_1^2 \frac{1}{x} dx$ within an accuracy of 0.0001 using the Trapezoid method? Using Simpson's method?

25. Explain how the error estimate shows that Simpson's rule give the exact answer for the integral of any polynomial of degree at most three. Use Simpson's rule with $n = 2$ on the integral $\int_0^{10} x^3 - 3x^2 - 5dx$ and compute the integral as an example.

26. Solve the initial value problem by first writing your answer as a definite integral and then evaluating: $\frac{dy}{dx} = \left(x + \frac{1}{x}\right)^2$, $y(1) = 1$.

27. Explicitely compute a limit of Riemann sums to evaluate $\int_1^3 x^2\,dx$.

28. Explicitely compute a limit of Riemann sums to evaluate $\int_{-1}^2 2x - x^2\,dx$.

29. Find the value of c in the Mean Value Theorem for Integrals for $\int_0^3 x^2\,dx$.

30. Give the proof of the first version of the Fundamental Theorem of Calculus.

31. Give a proof of the second version of the Fundamental Theorem of Calculus. You may use the first version.

32. True of False. There is a function whose derivative is $\sin(x^2)$. Either give a formula for the function, explain how you know there is such a function, or explain how you know there isn't.