

1. Compute  $\frac{d}{dx}(x^2 \sin(2x))$
2. Compute  $\frac{d}{dx} \frac{x^2+1}{x^2+x-4}$
3. Compute  $\frac{d}{dx}(\tan x + 3)^3$
4. Compute  $\frac{d}{dx} \sqrt[5]{(x^3 + 2)^2}$
5. Compute  $\frac{d}{dx} \frac{\sin x}{1+\sec x}$
6. Compute  $\frac{d}{dx} \frac{\csc^2 x + 3}{\pi}$
7. Use the definition of derivative to compute  $\frac{d}{dx} \sqrt{2x + 3}$
8. Use the definition of derivative to compute  $\frac{d}{dx} \frac{2x + 3}{x - 1}$
9. Derive the formula for the derivative of  $\sec x$ . You may use the formula for the derivative of  $\sin x$  and  $\cos x$ .
10. Compute the limit and use the definition of limit to prove it is what you say:  
 $\lim_{x \rightarrow 4} (3x^2 + 2x + 1)$
11. Compute the limit and use the definition of limit to prove it is what you say:  
 $\lim_{x \rightarrow -1} \frac{2x + 3}{x - 1}$
12. Compute the limit and use the definition of limit to prove it is what you say:  
 $\lim_{t \rightarrow 5} (3t^3 - 2t^2 + 4)$
13. Prove:  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$ .
14. Prove:  $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$ . (You may use Problem 12.)
15. Derive the formula for the derivative of  $\sin x$ . (You may use Problems 12 and 13.)
16. Derive the formula for the derivative of  $\cos x$  (You may use Problems 12 and 13.)
17. Use induction to prove that all polynomials are continuous.
18. Use the previous problem to prove that all rational functions are continuous on their domains.
19. Use induction to prove the  $\frac{d}{dx} x^n = nx^{n-1}$  for all positive integers  $n$ .
20. Use the fact that  $\frac{d}{dx} x^n = nx^{n-1}$  for any positive integer  $n$  to show that  $\frac{d}{dx} x^n = nx^{n-1}$  for any negative integer  $n$ .
21. Find an equation of the tangent line to  $y = \sin^3 x \cos x$  at the point where  $x = \frac{\pi}{4}$ .
22. Find an equation of the tangent line to  $y = x \sin x$  at the point where  $x = \frac{\pi}{3}$ .
23. Find an equation of the tangent line to  $y = \sqrt{3x - 2}$  at the point where  $x = 6$ .
24. Find equations for all the lines that pass through the point  $(2, 10)$  and are tangent to the graph of  $y = x^2 + 2x + 3$ . (Note that the point is not on the graph.)
25. Find equations for all the lines that pass through the point  $(1, 50/9)$  and are tangent to the graph of  $y = \frac{10}{x}$ . (Note that the point is not on the graph.)

26. Compute  $\lim_{x \rightarrow 0} \frac{\sin x + x}{x}$ .

27. Compute  $\lim_{x \rightarrow 0} \frac{\tan x}{x}$ .

28. Compute  $\lim_{\theta \rightarrow 0} \frac{1 - \cos^2 \theta}{\theta^2}$ .

29. Compute  $\lim_{t \rightarrow 0} t \csc t$ .

30. Compute  $\lim_{t \rightarrow 0} \frac{\sin(2t)}{3t}$ .

31. Prove  $\lim_{x \rightarrow 3} \sqrt{x^2 + 7} = 4$

32. Prove  $\lim_{x \rightarrow 2} \frac{5x^2 + 2x - 2}{x^2 - x - 4} = -11$

33. Let

$$f(x) = \begin{cases} \frac{\sin x}{x} & : x < 0 \\ \frac{\tan x}{x} & : 0 < x \\ 0 & : x = 0 \end{cases}$$

Is  $f$  continuous at  $x = 0$ ? If so show why. If not, determine if 0 is a removable discontinuity.

34. Give an example of a function that is continuous at  $x = 1$ , but it is not differentiable there.35. Prove that if  $f'(a)$  exists, then  $f(x)$  is continuous at  $x = a$ .36. Prove that  $\lim_{x \rightarrow 0} (3x + 2) \neq 3$  using the definition. (Do not use the fact that limits are unique.)37. Prove that  $\lim_{x \rightarrow 0} \frac{x+1}{x^2}$  does not exist.38. Prove that  $\lim_{x \rightarrow 4} \frac{x-4}{|x-4|}$  does not exist.39. State the definition of  $\lim_{x \rightarrow a} f(x) = l$  and state the negation of the definition.

40. Prove the product formula for derivatives.

41. Prove the quotient formula for derivatives.

42. Compute  $\lim_{x \rightarrow 3^+} f(x)$  and  $\lim_{x \rightarrow 3^-} f(x)$ . Does  $\lim_{x \rightarrow 3} f(x)$  exist? If so state how you know, if not state why.

$$f(x) = \begin{cases} \sin x & : x \leq 0 \\ x & : 0 < x \leq 3 \\ x^2 - 4 & : x > 3 \end{cases}$$

43. Compute  $\lim_{x \rightarrow 0^+} f(x)$  and  $\lim_{x \rightarrow 0^-} f(x)$ . Does  $\lim_{x \rightarrow 0} f(x)$  exist? If so state how you know, if not state why.

$$f(x) = \begin{cases} \sin x & : x \leq 0 \\ x & : 0 < x \leq 3 \\ x^2 - 4 & : x > 3 \end{cases}$$

44. Is there a number  $c$  that makes the function  $f$  continuous, where  $f$  is defined below? If so, find  $c$ .

$$f(x) = \begin{cases} x^2 + 3 & : x \leq 1 \\ -2x^2 + c & : x > 1 \end{cases}$$

45. Prove that if  $\lim_{x \rightarrow a} f(x) = l$  and  $\lim_{x \rightarrow a} g(x) = k$ , then  $\lim_{x \rightarrow a} (f(x) + g(x)) = l + k$ .46. Prove that if  $\lim_{x \rightarrow a} f(x) = l$  and  $c$  is a constant, then  $\lim_{x \rightarrow a} (cf(x)) = cl$ .