MATH 3000 Exam 1 February 18, 2011 Name____________________________________

You are not to use a calculator, book, notes, or others to do this exam.

1. Let $S \subset \mathbf{R}$ and $x \in S$. *(10 points)*
   a. Give the definition for x is a boundary point of the set S.

   b. Give the definition for x is an interior point of the set S.

2. Let $S \subset \mathbf{R}$. *(10 points)*
   a. Give the definition of S is open.

   b. Give the definition of S is closed.

3. Let A_k be a set for each $k \in I$ where I is some index set. *(8 points)*
   a. Give the definition of $\bigcup_{k \in I} A_k$.

   b. Give the definition of $\bigcap_{k \in I} A_k$.

4. Find all the boundary points for the sets. (No justification needed.) *(10 points)*

a. $[-2, 2)$

b. $\left\{ \dfrac{1}{n} \,\middle|\, n \in \mathbf{N} \right\}$

5. Find all interior points for the sets. (No justification needed.) *(10 points)*

a. $[-2, 2)$

b. $\left\{ \dfrac{1}{n} \,\middle|\, n \in \mathbf{N} \right\}$

6. Determine if each set is open, closed, both or neither. (No justification needed.) *(8 points)*

a. $(0, 1)$

b. $\left\{ \dfrac{1}{n} \,\middle|\, n \in \mathbf{N} \right\}$

c. $\left\{ \dfrac{1}{n} \,\middle|\, n \in \mathbf{N} \right\} \cup \{0\}$

d. $\emptyset$

7. Give the truth table for the statement $p \Rightarrow (p \wedge \sim q)$ *(8 points)*

8. Negate the statements *(10 points)*

   a. $\forall x, \exists y \ni x^2 < y$

   b. $\exists x \ni \forall x, x^2 < y$

9. You do not need to justify your answers to this problem. *(8 points)*

   a. Which is true, the statement in problem 8a), its negation, both or neither?

   b. Which is true, the statement in problem 8b), its negation, both or neither?

10. In class a theorem was proved concerning unions of open sets. Without using that theorem, prove that if U and V are both open sets, then $U \cup V$ is an open set. *(10 points)*

11. True or false. If true, just state that it is true. If false, state that it is false and then give an example that shows the statement is false. *(10 points)*

    a. Any union of open sets is open.

    b. Any intersection of open sets is open.

    c. If $S \subset \mathbf{R}$ and S is not open, then S is closed.