

Math 2510. Fill in the reasons

a) If $x+z = y+z$, then $x=y$.

pf: $x+z = y+z$

$$\therefore (x+z) + (-z) = (y+z) + (-z)$$

$$x + (z + (-z)) = y + (z + (-z))$$

$$x + 0 = y + 0$$

$$x = y.$$

□

b) If $x \in \mathbb{R}$, then $x \cdot 0 = 0$.

pf: $x \cdot 0 + x \cdot 0 = x(0+0)$

$$= x \cdot 0$$

$$= x \cdot 0 + 0$$

$$= 0 + x \cdot 0$$

$$\therefore x \cdot 0 = 0$$

□

c) If $x \in \mathbb{R}$, then $(-1)x = -x$

pf: $(-1)x + x = (-1)x + 1 \cdot x$

$$= (-1+1)x$$

$$= 0x$$

$$= 0$$

□

d) If $x, y \in \mathbb{R}$ and $xy = 0$, then $x=0$ or $y=0$

pf: We assume $xy=0$. If $x=0$, we are done. So we assume $x \neq 0$. Then

$$xy = 0$$

$$(xy) \frac{1}{x} = 0 \cdot \frac{1}{x}$$

$$= \frac{1}{x} \cdot 0$$

$$= 0.$$

$$\therefore 0 = (xy)^{\frac{1}{x}}$$

$$= x(y \cdot \frac{1}{x})$$

$$= x(\frac{1}{x}y)$$

$$= (x \cdot \frac{1}{x})y$$

$$= 1y$$

$$= y$$

$$\therefore y = 0$$

□

$$e) (-1)(-1) = 1$$

$$\begin{aligned} \text{pf: } (-1)(-1) + (-1)(1) &= -1(-1+1) \\ &= -1(0) \\ &= 0 \cdot (-1) \\ &= 0 \end{aligned}$$

$$\therefore (-1)(-1) + -1 = 0$$

$$\therefore (-1)(-1) + -1 = 1 + -1$$

$$\therefore (-1)(-1) = 1$$

□

$$f) 1 > 0$$

pf: Either $1 = 0$, $1 < 0$, or $1 > 0$.

$$1 \neq 0$$

$$\therefore 1 < 0 \text{ or } 1 > 0$$

If $1 < 0$, then $1 + (-1) < 0 + -1$

$$\therefore 0 < -1 + 0$$

$$\therefore 0 < -1$$

$$\therefore 0(-1) < (-1)(-1)$$

$$\therefore (-1)0 < (-1)(-1)$$

$$0 < (-1)(-1)$$

$$0 < 1$$

$$\therefore 1 > 0.$$

□

g) If $x \in \mathbb{R}$, then $x > 0$ iff $-x < 0$.

pf. (\Rightarrow) Suppose $x > 0$.

$$x + (-x) > -x$$

$$0 > -x$$

$$\therefore -x < 0.$$

(\Leftarrow) Suppose $-x < 0$.

$$-x + x < 0 + x$$

$$x + -x < 0 + x$$

$$0 < 0 + x$$

$$0 < x$$

$$\therefore x > 0.$$

□

h) If $a, b \in \mathbb{R}$, $a \neq 0$ and $b \neq 0$, then $\frac{1}{a} \cdot \frac{1}{b} = \frac{1}{ab}$.

pf. $(ab) \cdot \left(\frac{1}{a} \cdot \frac{1}{b}\right) = (ab) \cdot \frac{1}{a} \cdot \frac{1}{b}$
 $= (a(b \cdot \frac{1}{a})) \cdot \frac{1}{b}$
 $= ((a \cdot \frac{1}{a})b) \cdot \frac{1}{b}$

$$= (1 \cdot b) \frac{1}{b}$$

$$= b \cdot \frac{1}{b}$$

$$= 1$$

$$ab \neq 0.$$

$$\therefore ((ab) \cdot \left(\frac{1}{a} \cdot \frac{1}{b}\right)) \frac{1}{ab} = \frac{1}{ab}$$

$$(ab) \left(\left(\frac{1}{a} \cdot \frac{1}{b}\right) \left(\frac{1}{ab}\right)\right) = \frac{1}{ab}$$

$$(ab) \left(\frac{1}{ab}\right) \cdot \left(\frac{1}{a} \cdot \frac{1}{b}\right) = \frac{1}{ab}$$

$$((ab)(\frac{1}{ab})) \cdot (\frac{1}{a} \cdot \frac{1}{b}) = \frac{1}{ab}$$

$$1 \cdot (\frac{1}{a} \cdot \frac{1}{b}) = \frac{1}{ab}$$

$$(\frac{1}{a} \cdot \frac{1}{b}) \cdot 1 = \frac{1}{ab}$$

$$\frac{1}{a} \cdot \frac{1}{b} = \frac{1}{ab} \quad \square$$

i) If $a, b, c, d \in \mathbb{R}$ with $b \neq 0$ and $d \neq 0$, then

$$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$$

pf: $\frac{a}{b} + \frac{c}{d} = a \cdot \frac{1}{b} + c \cdot \frac{1}{d}$

$$= (a \cdot \frac{1}{b})(1) + (c \cdot \frac{1}{d})(1)$$

$$= (a \cdot \frac{1}{b})(d \cdot \frac{1}{d}) + (c \cdot \frac{1}{d})(b \cdot \frac{1}{b})$$

$$= ((a \cdot \frac{1}{b})d) \frac{1}{d} + ((c \cdot \frac{1}{d})b) \cdot \frac{1}{b}$$

$$= (a \cdot (\frac{1}{b} \cdot d)) \frac{1}{d} + (c(\frac{1}{d} \cdot b)) \frac{1}{b}$$

$$= (a(d \cdot \frac{1}{b})) \frac{1}{d} + (c(b \cdot \frac{1}{d})) \cdot \frac{1}{b}$$

$$= ((ad) \cdot \frac{1}{b}) \frac{1}{d} + ((cb) \cdot \frac{1}{d}) \cdot \frac{1}{b}$$

$$= (ad) \cdot (\frac{1}{b} \cdot \frac{1}{d}) + (cb) (\frac{1}{d} \frac{1}{b})$$

$$= (ad) \cdot (\frac{1}{bd}) + (cb) \cdot (\frac{1}{db})$$

$$= (ad) \cdot \frac{1}{bd} + (cb) \cdot \frac{1}{bd}$$

$$= \left(\frac{1}{bd}\right) \cdot (ad) + \left(\frac{1}{bd}\right)(cb)$$

$$= \frac{1}{bd} (ad + cb)$$

$$= (ad + cb) \cdot \left(\frac{1}{bd}\right)$$

$$= \frac{ad + cb}{bd}$$