MATH 3000 Exam 1 February 18, 2011 Name_____

You are not to use a calculator, book, notes, or others to do this exam.

1. Let $S \subset \mathbf{R}$ and $x \in S$.   *(10 points)*

   a. Give the definition for x is a boundary point of the set S.

   b. Give the definition for x is an interior point of the set S.

2. Let $S \subset \mathbf{R}$.   *(10 points)*

   a. Give the definition of S is open.

   b. Give the definition of S is closed.

3. Let A_k be a set for each $k \in I$ where I is some index set.   *(8 points)*

   a. Give the definition of $\bigcup_{k \in I} A_k$.

   b. Give the definition of $\bigcap_{k \in I} A_k$.

4. Find all the boundary points for the sets. (No justification needed.)   *(10 points)*
   a. $[-2, 2)$

   b. $\left\{ \dfrac{1}{n} \,\middle|\, n \in \mathbf{N} \right\}$

5. Find all interior points for the sets. (No justification needed.)   *(10 points)*
   a. $[-2, 2)$

   b. $\left\{ \dfrac{1}{n} \,\middle|\, n \in \mathbf{N} \right\}$

6. Determine if each set is open, closed, both or neither. (No justification needed.)   *(8 points)*
   a. $(0, 1)$

   b. $\left\{ \dfrac{1}{n} \,\middle|\, n \in \mathbf{N} \right\}$

   c. $\left\{ \dfrac{1}{n} \,\middle|\, n \in \mathbf{N} \right\} \cup \{0\}$

   d. $\emptyset$

7. Give the truth table for the statement $p \Rightarrow (p \wedge \sim q)$ (8 points)

8. Negate the statements   (10 points)
   a. $\forall x, \exists y \ni x^2 < y$

   b. $\exists x \ni \forall x, x^2 < y$

9. You do not need to justify your answers to this problem.   (8 points)
   a. Which is true, the statement in problem 8a), its negation, both or neither?

   b. Which is true, the statement in problem 8b), its negation, both or neither?

10. In class a theorem was proved concerning unions of open sets. Without using that theorem, prove that if U and V are both open sets, then $U \cup V$ is an open set. *(10 points)*

11. True or false. If true, just state that it is true. If false, state that it is false and then give an example that shows the statement is false. *(10 points)*
   a. Any union of open sets is open.

   b. Any intersection of open sets is open.

   c. If $S \subset \mathbf{R}$ and S is not open, then S is closed.