Purpose: To count the number of functions of various types.

Basic counting principle: If two experiments are done, the first having n possible outcomes and the second having m possible outcomes for each outcome of the first experiment, then there are a total of nm possible outcomes for the combination of both experiments.

Apply the basic counting principle to answer the following questions. It may be helpful to think of a function $f : A \to B$ as arrows from elements of A to elements of B.

1. How many functions are there $f : A \to B$ if A has exactly n elements and B has exactly m elements? To answer this question, do the following.
   a) List all the functions $f : \{1, 2\} \to \{1, 2\}$. How many are there?
   b) List all the functions $f : \{1, 2\} \to \{1, 2, 3\}$. How many are there?
   c) List all the functions $f : \{1, 2, 3\} \to \{1, 2\}$. How many are there?
   d) Make a conjecture about how many functions there are from a set with n elements to a set with m elements. You may wish to do a few more examples using various values of n and m to help you make your conjecture. Use the basic counting principle to verify that your answer is correct in general.

2. How many injective functions are there from a set with n elements to a set with m elements? To answer this questions, do the following.
   a) First answer the question when $n = 2$ and $m = 2$.
   b) Answer the question when $n = 2$ and $m = 3$.
   c) Answer the question when $n = 3$ and $m = 2$.
   d) Answer the question for some other choices of n and m and make a conjecture about how many injective functions there are.
   e) Use the basic counting principle to verify that your conjecture is correct in general.

3. Now, count how many functions $f : A \to B$ there are that are bijective if A has exactly n elements and B has exactly m elements. As before, try some examples for small values of n and m, make a conjecture and then verify your conjecture using the basic counting principle.

4. A room contains c chairs. Exactly p people enter the room and each sits in a chair. How many possible ways can the p people be seated? (No two people share a chair.) How is this problem related to the previous problems? Explain.

5. Look up Stirling numbers of the second kind and find out how they are related to the number of surjective functions between two sets.