Purpose

To estimate the time and height when a falling ball reaches terminal velocity.

Discussion

If there were no air resistance, a dropped ball would have a constant acceleration. Using the fact that the second derivative of position is acceleration, you can use calculus to determine the relation between distance dropped and time. If a ball is very dense and the drop is of reasonable height, then the force due to air resistance is negligible. This means that if you drop a dense ball and measure how far it dropped, you can determine the time it took the ball to fall with a pretty good degree of accuracy.

In this lab, you will drop a dense ball and a whiffle ball simultaneously and take a picture that shows their heights at the same instant. From the height of the dense ball, you can determine how long after the balls were dropped that the picture was taken. From this data, you can make a chart showing the time and distance for a falling whiffle ball. You can then fit the data to a model for a falling ball that accounts for air resistance and then estimate the terminal velocity and approximately when and where terminal velocity is achieved.

Required Equipment/Supplies

1. A long tape measure
2. A long strip of paper (at least 15 feet) with very visible markings at each foot (or 0.25 meters)
3. A dense ball such as a baseball or softball
4. A whiffle ball
5. At least one digital camera
6. Post-its
7. Foam pads (optional)
8. A laptop computer (optional)

The Model

There are various models for how air resistance effects a falling ball. Here we will use the simple model that the force from air resistance is proportional to the velocity. This model yields the differential equation

$$y''(t) = g - ky'(t) \tag{1}$$

where y is the distance the ball dropped, g is the acceleration due to gravity (with no air resistance) and k is a constant to be determined. When you take differential equations, you will be able to show that the solution to this differential equation is

$$y(t) = \frac{g}{k}t - \frac{g}{k^2}\left(1 - e^{-kt}\right), \tag{2}$$

as long as the initial velocity is 0 (meaning that you drop the ball and don't throw it).

Collecting Data

1. Find locations where you can safely drop objects at heights varying from approximately 10 feet to approximately 40 feet. You should get at least four different drop heights.

2. Secure the long strip of paper in such a way that the falling balls fall as close to the paper as possible. Measure and use a post-it note to mark the distance from the drop position to one of the marks on the paper. Be sure to write the distance very clearly, and be sure to update this distance from each drop height. (See attached picture.)

3. One person takes the two balls to the drop height, while the other team members are at the bottom. **Safety is of paramount importance.** Be sure that "all is clear" immediately prior to dropping the balls. You'll probably want to develop a system — other than yelling — so the people near the bottom of the drop can signal the person dropping the balls that it's safe to proceed.

4. The team member at the top will drop the dense ball and the whiffle ball simultaneously. At least one other team member, standing near the bottom of the drop, will attempt to take a picture before either ball hits the floor, but while both balls and the post-it note are in the field of view.

   Timing the picture may take some practice. If more than one student has a camera, try taking multiple pictures for each drop.

   Get at least two pictures from each drop height. It may be possible to do this with a single drop if at least two team members have digital cameras.

5. **Beware:** The dense ball may bounce a lot. Be sure to take the picture before the bounce! Also, you may want to try to land the balls on foam pads to reduce the size of the bounce.

6. After completing this experiment from at least four different drop heights, read the distance that the dense ball and the whiffle ball have dropped. You may need to upload your digital photographs to a computer, or you may be able to use a magnification feature on your digital camera. Use the markings on the strip of paper and the recorded height on the post-it to estimate the distance each ball dropped. Do your best to estimate this distance to one decimal point.

7. Enter the heights of the two balls in the first and third rows of the following table.

Dense Ball Position								
Time								
Whiffle Ball Position								

Analyzing Data

1. To begin, you will need to compute the time the balls were dropping for each drop. To do this, you will first solve the following initial-value problem* for the distance traveled by the dense ball (assuming that air resistance is negligible for it):

$$\begin{aligned} f''(t) &= g \\ f'(0) &= 0 \\ f(0) &= 0 \end{aligned} \tag{3}$$

   Solve for $f(t)$, the distance the dense ball drops at time t. Use this formula and the dense-ball positions to find the times in the above table. Remember that $g = 32$ ft/s$^2 = 9.81$ m/s^2.

2. Enter the times and the whiffle ball drop heights in a table in your calculator, and plot the data.

3. The model for the whiffle-ball data was given by (2). Compute $y'(t)$ and $y''(t)$, remembering that both g and k are constants. Use these to show that y satisfies the differential equation (1).

4. Use your calculator to plot the function (2) for various values of k along with the time/whiffle-ball data. You may want to start with $k = 1$ and then try slightly larger or smaller values of k to see what happens to the graph. Experiment to determine the value of k, accurate to one decimal place, that gives you the best fit. (This is a form of "visual" regression.)

5. Using the value of k you found, compute

$$\lim_{t\to\infty} y'(t)$$

   to determine the terminal velocity. How close to the terminal velocity was the whiffle ball for the longest drop time recorded by your team?

6. For what value of t does the whiffle ball attain a velocity within 1 foot per second (or 1/4 meter per second if you use metric) of its terminal velocity? How far would the whiffle ball drop at this time?

*You may notice that the right-hand side of (3) is different than the right-hand side of (1) since air resistance is assumed to be negligible