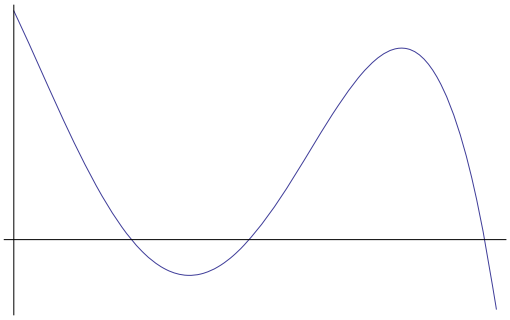
1. The graph below gives the velocity of an object as a function of time. *15 Points*



a) Sketch the graph of the acceleration as a function of time.

b) Sketch the graph of the position as a function of time.

2. Let A and B be sets and assume that A has exactly 5 elements and B has exactly 4 elements. *15 Points*

a) How many functions $f : A \to B$ are there?

b) How many injective functions $f : A \to B$ are there?

c) How many injective functions $f : B \to A$ are there?

3. Each of the tables is a function of the type indicated. Fill in the missing numbers using what you know about function patterns. *25 Points*

a) Exponential

x	$f(x)$
2	2
4	6
8	
16	

b) Power

x	$f(x)$
2	2
4	6
8	
16	

c) Linear

x	$f(x)$
2	2
4	6
8	
16	

d) Logarithmic

x	$f(x)$
2	2
4	6
8	
16	

4. **Use difference tables** to find an explicit formula for the $n^{\mathbf{th}}$ term in a sequence that starts *15 Points*

$$7, 21, 41, 67, 99, 137, \ldots$$

Show your work. No credit unless you show how you are using difference tables.

5. A car driving on the interstate highway does not have a speedometer. The driver tries to drive at a constant rate, but he has no cruise control and traffic makes him slow down or speed up occasionally. The following data gives a chart of the distance d the car traveled measured in miles at various times t measured in hours. *20 Points*

t	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
d	6.5	13.1	21.2	27.7	34.0	41.2	47.8	54.7	60.9	67.5

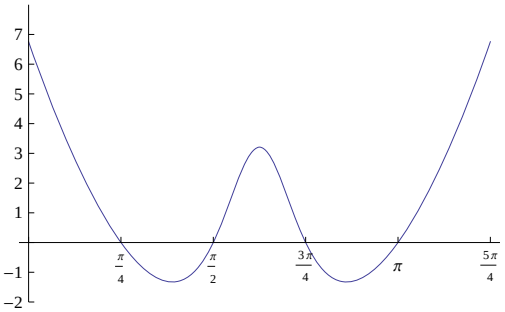
a) Based on the situation, what type of function should be used to model the data? Explain. (I am not asking you to do a regression in this part, that is part b.)

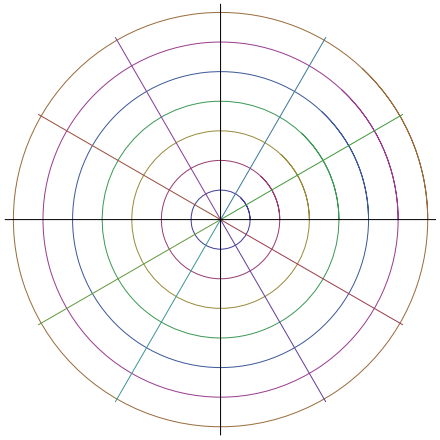
b) Based on your answer to part a), do a regression to find a function that fits the data. Be sure to actually write the function.

c) Based on your regression, what is a good estimate of the speed the car is moving.

d) Does the function in part b) give a good approximation to the actual data? Explain why or why not based on the regression.

6. The graph below is the graph of a function $y = f(x)$. Based on this graph, sketch a polar coordinate graph of the function $r = f(\theta)$. *15 Points*





7. State what it means for a function to be injective (or one-to-one). Do not mention a horizontal or vertical line test. *10 Points*

8. Four points on the graph of $f(x) = ax^3 + bx^2 + cx + d$ are $(0, 7), (2, 37), (4, 243), (6, 769)$. *20 Points*

   a) Write a matrix equation whose solution determines the coefficients a, b, c, d.

   b) Solve the matrix equation and give the values for a, b, c, d.

9. The parametric equations below define a curve in the plane. *15 Points*

$$\begin{aligned} x(t) &= 5 + 6\sin t \\ y(t) &= 3 - 2\cos^2 t \end{aligned}$$

a) Find an equation involving only x and y whose graph contains the graph of the parametrically defined curve.

b) What is the name of this graph?

c) Is the graph of the parametric equations exactly the same as the graph of the function you found in part a)? Explain why or why not.

10. A curve in the plane has equation $9x^2 + 4y^2 - 18x + 16y = 11$. *15 Points*

a) Rewrite the equation so it has a standard form of some curve we studied in class.

b) Based on your answer to part a), sketch a graph of the curve and state what kind of curve it is.

11. Convert the rectangular coordinates to polar coordiantes. *15 Points*

a) $(5, 7)$

b) $(-3, 2)$

c) $(-4, -6)$

d) $(0, -4)$

12. Three weights are balanced on a vector table. One weight has a mass of 42 g and it is located at an angle of 47°. The second one has a mass of 36 g at an angle of 302°. Find the mass and the angle for the third weight. *15 Points*

14. Have a wonderful summer! *5 Points*

MATH 2100 Exam 3 April 25, 2012 Name_____

1. Consider the system of equations below. *(10 points)*

$$2x + 3y - z = 13$$

$$4x + 2y + 3z = 4$$

$$x - y + 4z = -10$$

a. Rewrite the system of equations as one matrix equation.

b. Solve the matrix equation. (You may use a calculator.)

2. A circle passes through the points $(8, 22)$, $(3, 23)$, and $(-9, 5)$. *(10 points)*

a. Write a matrix equation whose solution will give an equation of the circle.

b. Solve the matrix equation in part a) and write an equation for the circle.

c. Rewrite the equation you found in part b) in standard form and then give the radius and the center of the circle.

3. A parabola with axis parallel with the y-axis passes through the points $(-3, 63)$, $(2, 13)$, and $(5, 55)$. *(10 points)*

   a. Write a matrix equation whose solution will give an equation of the parabola.

   b. Solve the matrix equation in part a) and write an equation of the parabola. You may let the calculator do the work for you.

   c. Rewrite the equation of the parabola from part b) in the form $y = a(x - h)^2 + k$.

4. Resolve each of the vectors into their x and y components. The angle is measured in the standard way from the $x-$axis and in a counterclockwise direction. Warning: If you use a calculator, be sure to have it set for degrees! *(6 points)*

   a. The magnitude of the vector is 12 and its direction is 42 degrees.

   b. The magnitude of the vector is 105 and its direction is 222 degrees.

5. The resolution of vectors into their x and y components are given. Give the magnitude of the vector and its direction measured from the $x-$axis. (Measured counterclockwise in the standard fashion. You can use radians or degrees, but be sure to say which you are using.) *(9 points)*

   a. $F_x = 5$ and $F_y = 13$

   b. $F_x = -7$ and $F_y = 14$

   c. $F_x = -7$ and $F_y = -3$

6. An object moves in the plane according to the parametric equations written below (t is time). *(10 points)*

$$x = 3 + 5\cos\left(\frac{\pi t}{8}\right) \quad y = 7 + 12\sin\left(\frac{\pi t}{8}\right)$$

   a. Find the position of the object at time $t = 2$.

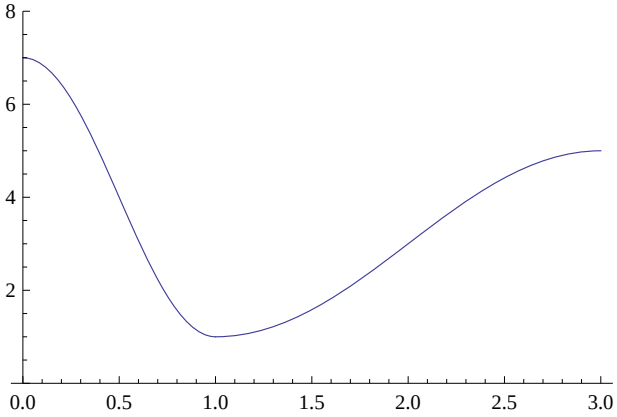
   b. Find the slope of the tangent line to the path of the object at the point you found in part a).

   c. Find an equation involving only x and y (no t) whose graph is the path of the object.

   d. Based on your answer to part c), what kind of curve is the path of the object?

7. In a three way tug of war, each team is pulling a rope away from the center where the three ropes are tied together. The three teams are in equilibrium, meaning that they are all pulling, but nothing is moving. One team is pulling with a force of 300 Newtons and its direction is at 42 degrees. A second is pulling with a force of 350 Newtons at an angle of 327 degrees. Find the force and direction that the third team is pulling. *(10 points)*

8. Find a piecewise defined function whose graph is as close to the following as you can make it. In particular, make the tangent lines horizontal at (0,7), (1,1), and (3,5), make it continuous and make the first derivative continuous. *(10 points)*



9. Find the rectangular coordinates of the points whose polar coordinates are given. (Recall that angles are measured in radians.) *(6 points)*

   a. $\langle 2, -4 \rangle$

   b. $\langle -4, 2 \rangle$

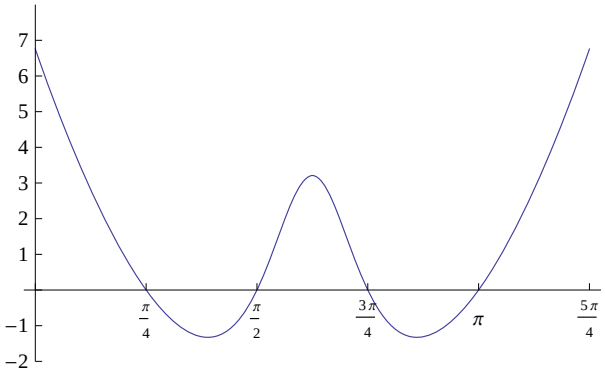
10. Find polar coordinates of the points whose rectangular coordinates are given. (Be sure to express your angles in radians.) *(9 points)*

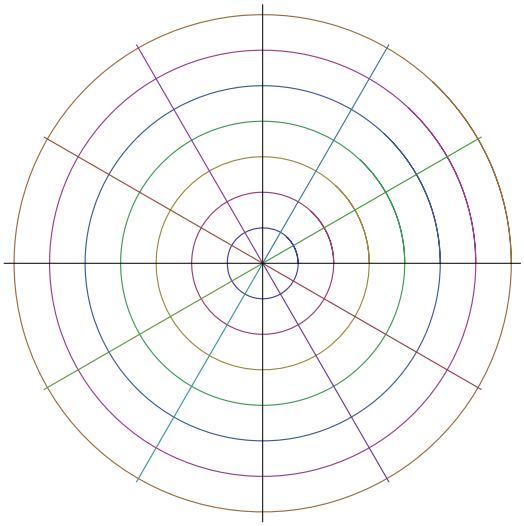
    a. $(2, -5)$

    b. $(-2, 5)$

    d. $(0, -2)$

11. The graph below is the graph of a function $y = f(x)$. Based on this graph, sketch a polar coordinate graph of the function $r = f(\theta)$. *(10 points)*





1. Each of the tables gives exact values of a function that is either linear, quadratic, a power function, an exponential function, or a logarithmic function. Use function patterns to identify the type of each function and find a specific function that fits the data. **Do not use regression to find the function.** Show work. *25 Points*

a)

x	$f(x)$
6	12
8	48
10	192
12	768
14	3072
16	12,288
18	49,152
20	19,6608

b)

x	$f(x)$
1	5
4	11
7	17
10	23
13	29
16	35

c)

x	$f(x)$
4	4
36	12
324	36
2916	108
26,244	324
236,196	972

d)

x	$f(x)$
4	4
12	36
36	324
108	2916
324	26,244
972	236,196

e)

x	$f(x)$
1	-7
5	-1
9	9
13	23
17	41
21	63
25	89
29	119

2. If n is the number in the left column of the table below, then the right column gives 10^n divdied by the number of prime numbers less than 10^n. For example, the first row has a 1 in the first column, so the entry in the second column is $\frac{10^1}{4}$ since there are 4 prime numbers less than 10 (2,3,5,7). *15 Points*

x	$f(x)$
1	2.5
2	4
3	5.952
4	8.137
5	10.425
6	12.740
7	15.047
8	17.357

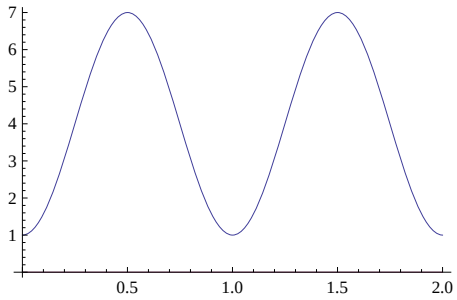
a) Based on function patterns, explain why it is reasonable to use linear regression to fit this data?

b) Give the equation of the regression line.

c) Explain how the R^2 value and the residue graph can be used to decide if a regression gives a good fit of the data. Based on these observations, do you think that the line you gave in part a) is a good model for the data? Explain.

3. When doing linear regression, you start with data and find a line to "fit" the data. Explain which line linear regression gives you. That is, among all possible lines, the regression line gives you a particular one. State clearly the property that determines the line. *10 Points*

4. Find a formula for the function whose graph is below. *15 Points*



5. John does an experiment and plots his data on semilog paper (log scale on the $y-$axis and a regular scale on the $x-$axis. He notices that his data lies on a line. Based on this observation, what type of function should John use to model his data? Show the derivation to justify your answer. *10 Points*

6. Find a an expicit formula for a sequence that starts with the numbers $8, 30, 82, 176, 324, 538, 830, 1212, \ldots$ *15 Points*
Use difference tables and show the rows of the tables you use.

7. True or False. If true, just say it is true. If false, say it is false and then explain why. *12 Points*

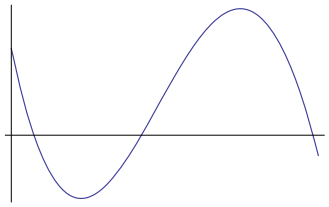
a) The R^2 value using a quadratic regression is never closer to 1 than the R^2 value using cubic regression.

b) Every sequence of numbers has a definite pattern.

c) When determining which model to use for measured data, it is best to ignore where the data came from, try every regression and then pick the one with the best R^2 value.

MATH 2100 Exam 1 February 15, 2012 Name_____

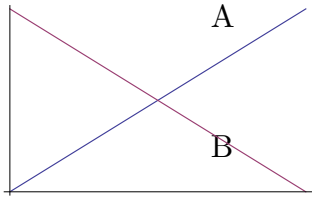
1. The graph below is a sketch of the graph of velocity as a function of time. *(10 points)*



a. Sketch the graph of acceleration verses time.

b. Sketch a graph of position verses time.

2. Two cars start at rest and their acceleration graphs are given below. Which car moved a greater distance, A or B? Explain. (Hint: First determine the velocity graph.) *(10 points)*



3. Consider functions $f : A \to B$ where A has exactly 5 elements. *(15 points)*

   a. If B has exactly 10 elements, then how many functions are there from A to B?.

   b. How many injective functions are there from A to B, again assuming that B has exactly 10 elements?

   c. How many bijective functions are there from A to B, this time assuming that B has exactly 5 elements?

   d. How many injective functions are there from A to B, this time assuming that B has exactly 3 elements?

4. State what it means for a function to be injective (or one-to-one). Do not mention a horizontal or vertical line test. *(10 points)*

5. Let the function $f : \mathbf{C} \to \mathbf{C}$ be given by the formula $f(x) = x^2 - 4x + 5$. *(15 points)*

a. Find the roots of f (including complex roots).

b. Find all the numbers x (including complex numbers) where $f(x)$ is a real number.

6. *(15 points)*

a. Derive the standard equation of a parabola with focus $(0, p)$ and directrix $y = -p$.

b. Write the standard form of the equation of a parabola with vertex (a, b), focus $(a, b+p)$ and directrix $y = b-p$. (You need not show the derivation, just the equation is enough.)

7. The graph of the equation $x^2 + 4y^2 - 2x = 3$ is an ellipse. *(10 points)*

a. Find the center of the ellipse.

b. Find the length of the major axis.

c. Find the length of the minor axis.

d. Find the foci.

8. How many different orders are possible if seven books are lined up on a shelf? *(5 points)*

9. True or false. If true, just say it is true. If false, say it is false and then either give an example that shows it is false or explain why. *(15 points)*

b. If $f : A \to B$ and $f(a_1) = f(a_2)$, then $a_1 = a_2$.

a. If $f : A \to B$ and $a_1 = a_2$, then $f(a_1) = f(a_2)$.

c. If $f : A \to B$ is a function, $a_1, a_2 \in A$, and $a_1 \neq a_2$, then $f(a_1) \neq f(a_2)$.

d. If $f : A \to B$, then the domain of f is all of A.

e. If $f : A \to B$, then the range of f is all of B.