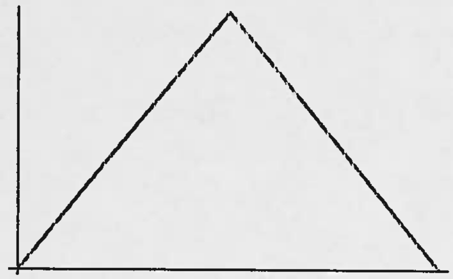
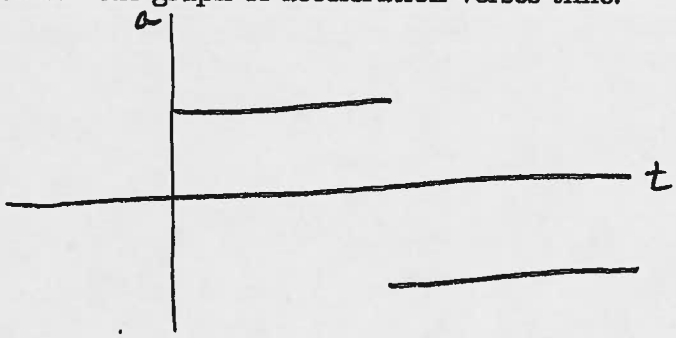
MATH 2100 Exam 1 October 1, 2012 Name______________________________

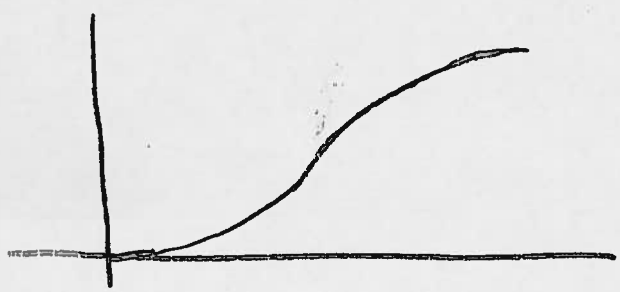
1. The graph below is a sketch of the graph of velocity as a function of time. *(10 points)*



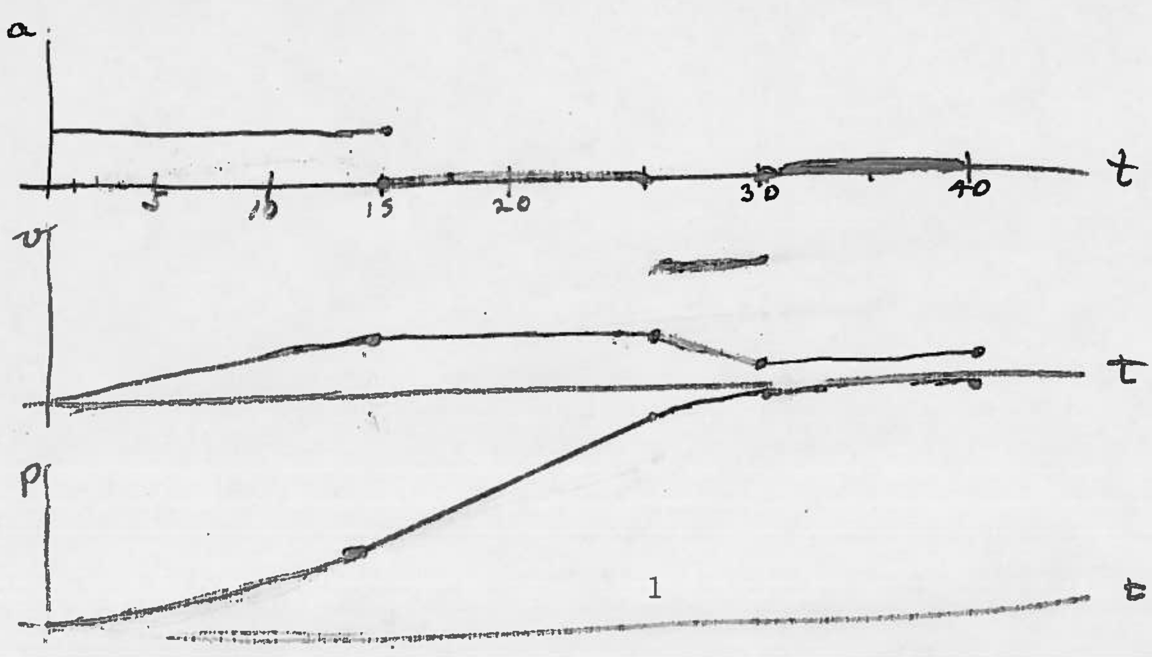
a. Sketch the graph of acceleration verses time.



b. Sketch a graph of position verses time.



2. A car starts at rest and accelerates with a constant positive acceleration for 15 seconds. It then travels at a constant velocity for 10 seconds. Then, the car slams on its brakes for 5 seconds with a constant acceleration until it cuts its speed in half. It then travels on at a constant rate for 10 more seconds. Sketch the graphs of the position, velocity and acceleration. *(10 points)*



3. Consider functions $f : A \to B$ where A has exactly 4 elements. *(15 points)*

a. If B has exactly 6 elements, then how many functions are there from A to B?

6^4

b. How many injective functions are there from A to B, again assuming that B has exactly 6 elements?

$6 \cdot 5 \cdot 4 \cdot 3$

c. How many bijective functions are there from A to B, this time assuming that B has exactly 4 elements?

$4!$

d. How many injective functions are there from A to B, this time assuming that B has exactly 3 elements?

0

4. Suppose that for every value of x in the set A there is exactly one corresponding value of y in B. Is this correspondance an injective function? Explain why or why not. *(10 points)*

No. $f(x)=x^2$ has this property but $f(x)=x^2$ is not injective.

5. Let the function $f : \mathbf{C} \to \mathbf{C}$ be given by the formula $f(x) = x^2 + 4x$. *(15 points)*

a. Find all the numbers x (including complex numbers) where $f(x)$ is a real number.

$x = a + bi$

$f(x) = f(a+bi) = (a+bi)^2 + 4(a+bi) = a^2 + 2abi - b^2 + 4a + 4bi$

This is a real number iff $2ab + 4b = 0$, $2b(a+2) = 0$

$\therefore f(a+bi)$ is real iff $b = 0$ or $a = -2$

b. Find all the numbers x (including complex numbers) where $f(x)$ is an imaginary number. (Recall that a number is imaginary if it has the form bi for some real number b.)

$f(a+bi) = a^2 + 2abi - b^2 + 4a + 4bi$ is imaginary iff

$a^2 - b^2 + 4a = 0$

c. If you graphed all the complex numbers in your answer to part b) you should get a curve in the complex number plane. What is the name of the curve in part b)?

$(a+2)^2 - b^2 = 4$

$\frac{(a+2)^2}{4} - \frac{b^2}{4} = 1$ $\therefore$ Hyperbola.

6. The graph of the function $f(x) = x^2 + 8x - 10$ is a parabola. *(10 points)*

a. Find the vertex of the parabola.

$x = \frac{-b}{2a} = \frac{-8}{2} = -4$ $y = (-4)^2 + 8(-4) - 10$
$= 16 - 32 - 10 = -26$

vertex is $(-4, -26)$

$f(x) = (x+4)^2 - 26 = \frac{(x+4)^2}{4p} - 26$ for $p = \frac{1}{4}$

b. Find the focus. $(-4, -26 + p) = (-4, -26 + \frac{1}{4}) = (-4, -\frac{103}{4})$

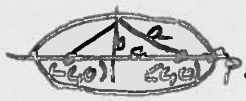
c. Find the directrix. $y = -26 - \frac{1}{4} = -\frac{105}{4}$.

7. An ellipse has foci $(-c, 0)$ and $(c, 0)$ and passes through the points $(a, 0)$ and $(0, b)$. *(15 points)*

a. Give the definition of ellipse.

The set of all points P in the plane such that the distance from P to $(c,0)$ plus the distance from $(-c,0)$ to P is $2a$.

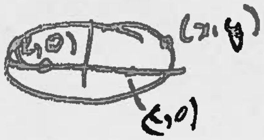
b. Use the definition of ellipse to find the distance sum referred to in the definition and to find the relation between a, b, c.



The distance is $(a-c)+(a-(-c)) = 2a$.

$a^2 = b^2 + c^2$

b. Show the derivation of the equation of the ellipse. Simplify your answer and leave it in standard form.



$$\sqrt{(x-c)^2+y^2} + \sqrt{(x+c)^2+y^2} = 2a$$

$$(x-c)^2+y^2 = \left(2a - \sqrt{(x+c)^2+y^2}\right)^2$$

$$(x-c)^2+y^2 = 4a^2 - 4a\sqrt{(x+c)^2+y^2} + (x+c)^2+y^2$$

$$(x-c)^2-(x+c)^2-4a^2 = -4a\sqrt{(x+c)^2+y^2}$$

$$-4cx-4a^2 = -4a\sqrt{(x+c)^2+y^2}$$

$$cx+a^2 = a\sqrt{(x+c)^2+y^2}$$

$$(cx+a^2)^2 = a^2\left[(x+c)^2+y^2\right]$$

$$c^2x^2+2ca^2x+a^4 = a^2(x^2+2xc+c^2+y^2)$$

$$c^2x^2+2ca^2x+a^4 = a^2x^2+2ca^2x+a^2c^2+a^2y^2$$

$$(c^2-a^2)x^2 - a^2y^2 = a^2c^2 - a^4$$

$$(a^2-c^2)x^2 + a^2y^2 = a^2(a^2-c^2)$$

$$\frac{x^2}{a^2} + \frac{y^2}{a^2-c^2} = 1 \qquad \therefore \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

8. In a class of 16 students, how many different ways could the instructor assign one letter grade (A,B,C,D, or F) to each student? *(5 points)*

5^{16}

9. True or false. If true, just say it is true. If false, say it is false and then either give an example that shows it is false or explain why. *(15 points)*

b. If $f : A \to B$ and $f(a_1) = f(a_2)$, then $a_1 = a_2$.

False $f(x) = x^2$ $f : \mathbb{R} \to \mathbb{R}$.

$a_1 = 1,\ a_2 = -1$.

a. To say that $f : A \to B$ is injective means that if and $a_1, a_2 \in A$ and $a_1 \neq a_2$, then $f(a_1) \neq f(a_2)$.

True

c. If two experiments are performed and there are n possible outcomes for the first experiment and for each outcome of the first, there are m possible outcomes for the second, then there are $n + m$ possible outcomes for the combined experiments.

F $n \cdot m$

d. If $f : A \to B$, then the domain of f is all of A.

T

e. If $f : A \to B$, then the range of f is all of B.

F $f : \mathbb{R} \to \mathbb{R}$

$f(x) = x^2$

range $= \{x \in \mathbb{R} \mid x \geq 0\}$.

Math 2100 Exam 2 November 12, 2012 Name:

1. Each of the tables gives exact values of a function that is either linear, quadratic, a power function, an exponential function, or a logarithmic function. Use function patterns to identify the type of each function and find a specific function that fits the data. **Do not use regression to find the function.** Show work. *20 Points*

a)

x	$f(x)$
1	5
4	14
7	23
10	32
13	41
16	50
19	59

add-add ∴ linear

$y = 3x + 2$

b)

x	$f(x)$
3	2
5	18
7	162
9	1458
11	13122

add-mult.

∴ exponential

$y = \frac{2}{27} 3^x$

c)

x	$f(x)$
10000	1
1000	2
100	4
10	8
1	16

mult-mult

∴ power

$y = 16 x^{-\log_{10} 2}$

d)

x	$f(x)$
3	-4
5	1
7	12
9	29
11	52
13	81

add- 2^{ND} diff

∴ quad.

$y = \frac{3}{4} x^2 - \frac{7}{2} x - 25$

e)

x	$f(x)$
2	3
18	5
162	7
1458	9
13122	11

mult - add

∴ log

$$y = \frac{\ln \frac{27}{2}}{\ln 3} + \frac{\ln x}{\ln 3}$$

2. A student on a distant planet is trying to determine the acceleration due to gravity on her planet. The table below shows data she collected on the time it takes a heavy object to fall. Curiously, they use seconds for time and meters for distance on their planet just like we Earthlings do! *10 Points*

Time	Distance
1	3.7
1.5	8.3
2	14.5
2.5	22.8
3	32.9
3.5	44.65
4	58.1
4.5	73.9

a) What type of regression should the alien use? Explain why.

$D=\frac{1}{2}gt^2$ $\therefore$ use either quadratic or power.

b) Use the regression that you answered in part a) and give the regression equation.

either is ok

quad gives $D=3.66t^2-.098t+.15$ $R^2=.99998\ldots$

power gives $D=3.69t^{1.9898}$ $R^2=.999977$

c) Based on your answer to part b), what would you estimate as the acceleration due to gravity on the far away planet?

quad: $2(3.66)=7.32$ m/sec²

power: $2(3.69)=7.38$ m/sec²

d) Explain generally how the R^2 value and the residue graph can be used to decide if a regression gives a good fit to data. Do you think that the equation you gave in part b) is a good model for the data? Explain.

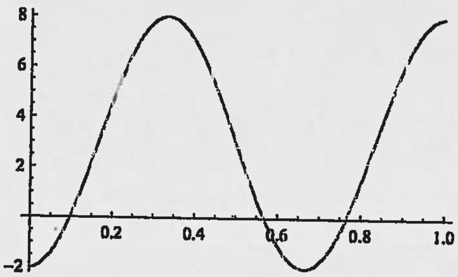
For a good fit R^2 is close to 1 and the residual plot is random.

This is true for the regression in part b.

3. When doing linear regression, you start with data and find a line to "fit" the data. Explain which line linear regression gives you. That is, among all possible lines, the regression line gives you a particular one. State clearly the property that determines the line. *5 Points*

The sum of the squares of the residuals is minimized over all possible linear functions.

4. Find a formula for the function whose graph is below. *10 Points*



$y = 3 + 5\cos(8.976(x - .3))$

5. John does an experiment and plots his data on log-log paper (log scale on both axes). He notices that his data lies on a line. Based on this observation, what type of function should John use to model his data? Show the derivation to justify your answer. *10 Points*

$\log y = m \log x + b$

$y = 10^{m \log x + b} = 10^{(\log x)\cdot m} \cdot 10^b = 10^b \cdot (10^{\log x})^m$

$= 10^b x^m = cx^m$ where c is the constant 10^b.

$\therefore$ power function.

6. Show that if $f(x)$ is an exponential function ($f(x) = ab^x$), then when a constant c is added to x, then the value of $f(x)$ is multiplied by a constant. What is the constant that $f(x)$ is multiplied by? *10 Points*

Let x_1 be a number

Let $x_2 = x_1 + c$

$\therefore f(x_1) = ab^{x_1}$ and $f(x_2) = ab^{x_2} = ab^{x_1 + c} = ab^{x_1}b^c$

$= b^c ab^{x_1} = b^c f(x_1)$.

The constant is b^c.

7. Find an explicit formula for a sequence that starts with the numbers 10, 52 ,160, 364, 694, 1180, 1852, 2740,... Use difference tables and show the rows of the tables you use. *10 Points*

$$x_n = 5n^3 + 3n^2 - 2n + 4$$

8. Use linear equations and matrices to determine the equations requested below. Be sure to show the matrices that you are using. (You may use the calculator to do the matrix manipulations.) *10 Points*

a) A quadratic whose graph passes through the points $(2, 16)$, $(5, 130)$, and $(-2, 4)$. Put your answer in standard form.

$$A = \begin{bmatrix} 4 & 2 & 1 \\ 25 & 5 & 1 \\ 4 & -2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 16 \\ 130 \\ 4 \end{bmatrix}$$

$$A^{-1}B = \begin{bmatrix} 5 \\ 3 \\ -10 \end{bmatrix}$$

$$\therefore \; y = 5x^2 + 3x - 10$$
$$= 5\left(x + \tfrac{3}{10}\right)^2 - \frac{209}{20}$$

b) A circle whose graph passes though the points $(-2, -5)$, $(8, 19)$, $(3, 20)$. Put your answer in standard form and identify the center and radius of the circle.

$$A = \begin{bmatrix} -2 & -5 & 1 \\ 8 & 19 & 1 \\ 3 & 20 & 1 \end{bmatrix} \quad B = \begin{bmatrix} -29 \\ -425 \\ -409 \end{bmatrix}$$

$$A^{-1}B = \begin{bmatrix} -6 \\ -14 \\ -111 \end{bmatrix}$$

$$\therefore \; x^2 + y^2 - 6x - 14y - 111 = 0$$
$$(x-3)^2 + (y-7)^2 = 169$$

$\therefore$ center at $(3, 7)$
radius $= 13$.

9. True or False. If true, just say it is true. If false, say it is false and then explain why. *15 Points*

a) The R^2 value using a quadratic regression is never closer to 1 than the R^2 value using cubic regression.

T

b) Every sequence of numbers has a definite pattern.

F No pattern is assumed
a sequence is any function $\mathbb{N} \to \mathbb{R}$.

c) A sequence is a set of numbers.

F A set is different from a list or function. A sequence is a function $\mathbb{N} \to \mathbb{R}$.

d) The third difference of any quadratic function is 0.

T

e) When determining which model to use for measured data, it is best to try different regressions and see which gives the best R^2 value.

F
- Could fit data exactly and still be a bad model
- May want a simpler model
- The situation may suggest the model to use.

+ other possible reasons!

MATH 2100 Exam 3 December 3, 2012 Name________________________

You are not to use a book, notes, or others to do this exam. You may use a calculator. SHOW ALL WORK.

1. Consider the parametric equations $x(t) = 3t + 7$ and $y(t) = t^2 + 5$. *(20 points)*

a. Find $x'(t)$. 3

b. Find $y'(t)$. $2t$

c. Find the slope of the tangent line to the graph of $(x(t), y(t))$. Your answer should be a function of t.

$$m = \frac{2t}{3}$$

d. Eliminate the parameter t to find a relation between x and y for points on the graph of $(x(t), y(t))$.

$$x = 3t + 7$$
$$t = \frac{x-7}{3}$$
$$\therefore y = \left(\frac{x-7}{3}\right)^2 + 5$$

e. Find an equation of the tangent line to the graph of $(x(t), y(t))$ at the point where $t = 1$

at $t = 1$, $m = 2/3$, $x = 10$, $y = 6$

$$\therefore y - 6 = \frac{2}{3}(x - 10)$$

2. Let $x(t) = 7 + 3\cos(2t+3)$ and $y(t) = 3 + 4\sin(2t+3)$. *(15 points)*

a. Eliminate the parameter t to find a relation between x and y. Make your answer as simple as possible and put it in standard form for the type of curve it is. Show your work or you will receive no credit.

$$\frac{x-7}{3} = \cos(2t+3) \qquad \frac{y-3}{4} = \sin(2t+3)$$

$$\sin^2(2t+3) + \cos^2(2t+3) = 1$$

$$\therefore \left(\frac{y-3}{4}\right)^2 + \left(\frac{x-7}{3}\right)^2 = 1$$

$$\frac{(x-7)^2}{3^2} + \frac{(y-3)^2}{4^2} = 1$$

b. What is the name of the graph of $(x(t), y(t))$. Explain how you know your answer is correct. Ellipse. The equation is in standard form.

3 The rectangular coordinates of points are given below. For each one, find polar coordinates that represent the same point in the plane. *(15 points)*

a. $(1, -1)$ $\langle \sqrt{2}, -\pi/4 \rangle$

b. $(-1, 1)$ $\langle \sqrt{2}, \frac{3\pi}{4} \rangle$

c. $(0, -3)$ $\langle 3, \frac{3\pi}{2} \rangle$

d. $(-2, -4)$ $\langle \sqrt{20}, \pi + \arctan 2 \rangle$

4. The polar coordinates of points are given below. For each one, find rectangular coordinates that represent the same point in the plane. *(15 points)*

a. $\langle 2, \frac{\pi}{4} \rangle$

$(\sqrt{2}, \sqrt{2})$

b. $\langle -3, \frac{7\pi}{6} \rangle$

$\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$

c. $\langle 0, 3\pi \rangle$

$(0, 0)$

5. Find an equation involving x and y whose graph in rectangular coordinates is the same as the graph of $r = 4\cos\theta$ in polar coordinates. What is the name of this graph? *(10 points)*

$r^2 = 4r\cos\theta$

$x^2 + y^2 = 4x$

$x^2 - 4x + y^2 = 0$

$(x-2)^2 + y^2 = 4$

circle center $(2, 0)$
radius 2.

6. Find an equation in polar coordinates whose graph is the same as the graph of the line $y = 3x + 2$ in rectangular coordinates.

(10 points)

$r\sin\theta = 3r\cos\theta + 2$

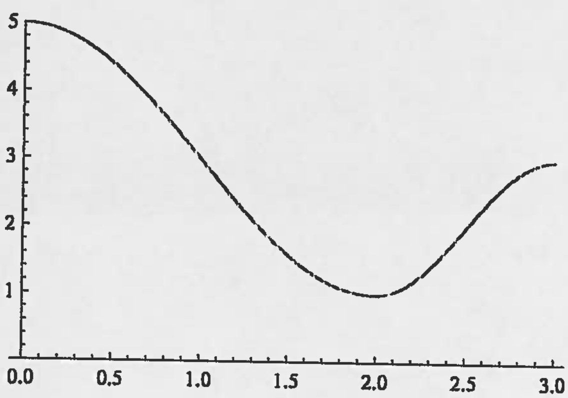
$r\sin\theta - 3r\cos\theta = 2$

$r(\sin\theta - 3\cos\theta) = 2$

$$r = \frac{2}{\sin\theta - 3\cos\theta}$$

7. Find a piecewise function whose graph is given below. Be sure to make the function have critical points as indicated in the graph.

(15 points)



$$f(x) = \begin{cases} 3 + 2\cos\frac{\pi}{2}x & 0 \le x \le 2 \\ 2 + \cos(\pi(x-3)) & 2 < x \le 3 \end{cases}$$

OR

$$f(x) = \begin{cases} -2x^2 + 5 & 0 \le x \le 1 \\ 2x^2 - 8x + 9 & 1 < x \le 2 \\ 4x^2 - 16x + 17 & 2 < x \le \frac{5}{2} \\ -4x^2 + 24x - 33 & \frac{5}{2} < x \le 3 \end{cases}$$