Homework: More on Conic Sections

1. Describe the symmetries of a parabola, ellipse, and a hyperbola. Explain why they have these symetries in two ways, first use the definition and geometry and then use the equations derived for each. (If you don't recall how to check a graph has symmetry about the $x-$axis, $y-$axis, or origin, look it up in your calculus book or online.)
2. Use geometry or algebra to show that all points on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ lie in the rectangle with $-a \leq x \leq a$ and $-b \leq x \leq b$. Use this fact and the symmetry of an ellipse to describe how you can quickly sketch the graph of an ellipse.
3. Use geometry or algebra to show that no point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is in the band with $-a < x < a$. What about the band $-b < y < b$? Why is there a difference?
4. Consider only the part of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ in the first quadrant. With this restriction, explain why we have a function. Now, solve for y to get a formula for this function $f(x)$. Recall that a graph of a function has an asymptote L if as x grows, the graphs of the function and the line L get close together. Show that the line $y = \frac{b}{a}x$ is an asymptote for $f(x)$ by computing the limit of their difference:
$$\lim_{x\longrightarrow\infty}\left(f(x) - \frac{b}{a}x\right).$$
Does the hyperbola approach the line from below the line or from above?
5. Use symmetry and the asymptote found in part 4) to describe a way to make a quick sketch of a hyperbola.
6. Determine the vertex of each parabola:
   a. $y = x^2 + 3x + 4$
   b. $y = 3x^2 + 6x - 4$
7. Find the center of each ellipse. Also determine the length of the major and minor axis.

a. $4x^2 + y^2 - 24x - 4y + 36 = 0$

b. $9x^2 + 4y^2 + 18x - 24y + 44 = 0$