

1. Determine if the series converges or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} 1.000001^n$$

2. Determine if the series converges or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n} + n + \sqrt{n}}$$

3. Determine if the series converges or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} \frac{1}{3n + 7}$$

4. Determine if the series converges or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} \frac{n^2 + 2n + 1}{10n^3 + 3n^2 + 4n - 1}$$

5. Determine if the series converges or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} (2n!)e^{-n/2}$$

6. Determine if the series converges or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} \frac{3}{\sqrt{n^2 + n + 3}}$$

7. Determine if the series converges or diverges. Justify carefully. $\sum_{n=1}^{\infty} \left(1 - \frac{1}{n}\right)^{n^2}$

8. Determine if the series converges or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n + 1)!}$$

9. Determine if the series converges or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} \tan\left(\frac{1}{n^2}\right)$$

10. Determine if the series converges absolutely, converges, or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} (-1)^n \frac{n^3}{3^n}$$

11. Determine if the series converges absolutely, converges, or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} (-1)^n (\sqrt[3]{n+1} - \sqrt[3]{n})$$

12. Determine if the series converges absolutely, converges, or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} \frac{\cos\left(\frac{n\pi}{2}\right)}{n}$$

13. Determine if the series converges absolutely, converges, or diverges. Justify carefully. $\sum_{n=1}^{\infty} \left(\frac{1}{n} - 1\right)^n$

14. Determine if the series converges absolutely, converges, or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$$

15. How many terms do you need to add to get a number that is within 0.0001 of the infinite sum for

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(4n+3)^3}$$

16. How many terms do you need to add to get a number that is within 0.01 of the infinite sum for

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n^2+7}$$

17. How many terms do you need to add to get a number that is within 0.0001 of the infinite sum for

$$\sum_{n=0}^{\infty} \frac{1}{3n+1}$$

Explain your answer.

18. Find the interval of convergence and the radius of convergence for

$$\sum_{n=0}^{\infty} \frac{\sqrt{n}x^n}{5^n}$$

19. Find the interval of convergence and the radius of convergence for

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$$

20. Find the interval of convergence and the radius of convergence for

$$\sum_{n=0}^{\infty} n^2 x^n$$

21. Find the interval of convergence and the radius of convergence for

$$\sum_{n=0}^{\infty} \frac{(x-2)^n}{n+1}$$

22. Find the interval of convergence and the radius of convergence for

$$\sum_{n=0}^{\infty} n! x^n$$

23. Find the interval of convergence and the radius of convergence for

$$\sum_{n=0}^{\infty} \frac{2n+1}{2^n+3^n} (x-1)^n$$

24. Find the power series for the function $\sqrt[3]{(1+x)^4}$ expanded about $x=0$.

25. Find the power series for the function $\sqrt[5]{(1+x)^6}$ expanded about $x=0$.

26. Find the power series for the function $\left(\frac{1}{x+2}\right)^3$ expanded about $x=0$.

27. Use the error term for the the Maclaurin series for e^x to estimate how close $1+x+x^2/2$ is to e^x if $-0.1 \leq x \leq 0.1$.

28. What degree Maclaurin polynomial does one need in order to approximate $\cos x$ to within 0.00005 for $0 \leq x \leq 0.5$?

29. What degree Maclaurin polynomial does one need in order to approximate e^x to within 0.000005 for $-1 \leq x \leq 1$?

30. Compute the derivative of the power series and compare the intervals of convergence of the series and its derivative.

$$\sum_{n=1}^{\infty} \frac{(x-2)^n}{n}$$

31. Compute the derivative of the power series and compare the intervals of convergence of the series and its derivative.

$$\sum_{n=0}^{\infty} \frac{x^n}{n!}$$

32. Find the third degree Maclaurin polynomial for the function $f(x) = \tan x$.

33. Find the fourth degree Taylor polynomial for the function $\ln x$ expanded about $x = 1$.

34. Derive the Taylor series for $\sin x$ expanded about $x = 0$.

35. Derive the Maclaurin series for e^x .

36. Use the error term for the the Maclaurin series for e^x to estimate how close $1+x+x^2/2$ is to e^x if $-0.1 \leq x \leq 0.1$.

37. What degree Maclaurin polynomial does one need in order to approximate $\cos x$ to within 0.00005 for $0 \leq x \leq 0.5$?

38. What degree Maclaurin polynomial does one need in order to approximate e^x to within 0.000005 for $-1 \leq x \leq 1$?

39. Derive the power series for $\arctan x$ (expand about $x = 0$). Use this to compute the sum of the series

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$$

40. Derive the Maclaurin series for $\ln(1+x)$. Use this to compute the sum of the series

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$$