

1. Compute the following:

$$\frac{d}{dx} e^x \ln(x)$$

2. Compute the following:

$$\frac{d}{dx} x^2 \arcsin x$$

3. Compute the following:

$$\frac{d}{dx} \frac{\arctan(x)}{1+x^2}$$

4. Compute the following:

$$\frac{d}{dx} \log_2(2x^2)$$

5. Compute the following:

$$\frac{d}{dx} \log_x 10$$

6. Compute the following:

$$\frac{d}{dx} 3^{2x+4}$$

7. Compute the following:

$$\frac{d}{dx} e^{x^2+3x-5}$$

8. Compute the following:

$$\frac{d}{dx} \left(x \operatorname{arcsec} \left(x^2/2 \right) \right)$$

9. Compute the derivatives and explain why you use the differentiation rules that you do.

a) $\frac{d}{dx} \sqrt{2^x}$

b) $\frac{d}{dx} x^{\sqrt{2}}$

10. Compute the following:

$$\int_{\pi/8}^{\pi/6} \tan(2x) dx$$

11. Compute the following:

$$\int \frac{3 \ln x^2}{x} dx$$

12. Compute the following:

$$\int \frac{1}{x \log_2 x} dx$$

13. Compute: $\int \cot(x) dx$

14. Compute the following:

$$\int \frac{\sec^2(3x)}{4 + \tan(3x)} dx$$

15. Compute the following:

$$\int \frac{e^x}{1+e^x} dx$$

16. Compute the following:

$$\int_{-\frac{1}{2} \ln 3}^0 \left(\frac{e^x}{1+e^{2x}} \right) dx$$

17. Compute the following:

$$\int 3^x dx$$

18. Compute the following:

$$\int \sin(x) e^{\cos(x)} dx$$

19. Compute the following:

$$\int \frac{1}{\sqrt{9-4x^2}} dx$$

20. Compute the following:

$$\int \frac{1}{2+x^2} dx$$

21. Compute the following:

$$\int x3^{x^2} dx$$

22. Compute the following:

$$\int \frac{1}{\sqrt{4x^2 - 9}} dx$$

23. Compute the following:

$$\int \frac{1}{1 - 16x^2} dx$$

24. Compute the following:

$$\int \frac{1}{x\sqrt{4x^2 - 1}} dx$$

25. Compute the following:

$$\int \frac{\arcsin(x)}{\sqrt{1 - x^2}} dx$$

26. Compute the following:

$$\int \frac{1}{3x + 2} dx$$

27. Compute $\int \frac{3}{9 - t^2} dt$

28. Compute $\int \frac{1}{t\sqrt{25 - t^2}} dt$

29. Compute $\int \frac{3}{t\sqrt{9 + t^2}} dt$

30. Compute the following:

$$\frac{d}{dx} \int_{e^{-x}}^{e^x} \ln t dt$$

31. Compute two different ways:

$$\frac{d}{dx} x^{2x}$$

32. Compute the following:

$$\frac{d}{dx} \log_x(x + 1)$$

33. Compute using logarithmic differentiation.

$$\frac{d}{dx} \sqrt[4]{\frac{(x^2 + 1)x^3}{(2x - 1)^3 \sqrt{x - 1}}}$$

34. Find an equation of the line tangent to $y = \arctan x$ at the point where $x = 1$.

35. Find an equation of the line tangent to the graph of $y = xe^{x^2}$ at the point where $x = 2$.

36. Find an equation for the line tangent to $y = x \ln x$ at the point where $x = e$.

37. Find the area bounded by the graphs of $y = e^x$ and $y = (e - 1)x + 1$.

38. Find the volume of the solid of revolution obtained by rotating the region bounded by the coordinate axes, $y = 3$, and $x = \frac{2}{\sqrt{y+1}}$ about the y -axis.

39. Find the absolute maximum and minimum values of the function $f(x) = e^x - 2x$ for $-1 \leq x \leq 2$.

40. Show that any curve of the form $y = -\frac{1}{2}x^2 + k$ and any curve of the form $y = \ln(x) + c$ intersect each other at right angles.

41. Find $\lim_{x \rightarrow 0} \frac{e^x - 1}{\sin x}$

42. Find $\lim_{x \rightarrow 0} \frac{\cos x - 1 + x^2/2}{x^4}$

43. Find $\lim_{n \rightarrow \infty} \left(1 - \frac{1}{n}\right)^{2n}$
44. Find $\lim_{n \rightarrow \infty} n^{\frac{1}{n}}$
45. Find $\lim_{n \rightarrow \infty} \frac{n^2}{1.00001^n}$
46. Find $\lim_{t \rightarrow 0^+} t \ln t$
47. Using the definition of \ln , prove that for any positive real numbers a and b , $\ln(ab) = \ln(a) + \ln(b)$.
48. Using the definition of \ln , prove that $\ln x$ is an increasing function.
49. Using the definition of the exponential function Exp , derive the formula for the derivative of $\text{Exp}(x)$.
50. Derive the formula for $\frac{d}{dx} \arctan x$.
51. Derive the formula for $\frac{d}{dx} \arccos x$.
52. Use the definition of $\log_a x$ and properties of \ln , prove that for any positive a, b, c , $\log_a(bc) = \log_a(b) + \log_a(c)$.
53. Use the definition of the exponential function $\text{Exp}(x)$, prove that $\text{Exp}(x + y) = \text{Exp}(x)\text{Exp}(y)$.
54. Let $a > 0$. Give the definition of a^x in terms of the exponential function. Use this definition (and properties of the exponential function) to show that $a^{x+y} = a^x a^y$ for any real numbers x and y .
55. Give the domain and range for each of the inverse trigonometric functions.
56. Prove that $\arcsin x + \arccos x = \frac{\pi}{2}$.
57. How many zeros does the function $f(x) = 2^x - 1 - x^2$ have on the real line? (Prove your answer.)
58. Find the formula for the n^{th} term of the sequence that starts
- $$1, 1, 3, 3, 5, 5, 7, 7, \dots$$
59. Find the formula for the n^{th} term of the sequence that starts
- $$0, 7, 0, 7, 0, 7, \dots$$
60. Find the formula for the n^{th} term of the sequence that starts
- $$1, \frac{1}{5}, \frac{1}{25}, \frac{1}{125}, \dots$$
61. Does the sequence below converge? If so find its limit.
- $$a_n = \sqrt{\frac{2n+3}{3n-1}}$$
62. Does the sequence below converge? If so find its limit.
- $$a_n = \frac{3n^2 + 2n - 1}{1 + 5n + 7n^2}$$
63. Does the sequence below converge? If so find its limit.
- $$a_n = (-1)^n + 1$$

64. Does the sequence below converge? If so find its limit.

$$b_n = \frac{n!}{n^n}$$

65. Does the sequence below converge? If so find its limit.

$$c_n = \sqrt[n]{n+1}$$

66. Determine if the sequence below is bounded above, bounded below, nondecreasing, or nonincreasing.

$$a_n = \frac{n+1}{2n+3}$$

67. Determine if the sequence below is bounded above, bounded below, nondecreasing, or nonincreasing.

$$a_n = \sin n + \frac{1}{n}$$

68. If 10g of a radioactive substance decays to 8g in 3 years, find its half life. If the half life of another substance is 1 year, find how long it takes 10g of the substance to decay to 8g.

69. Suppose that after 10 years of earning interest at the rate of 7% per year, you have \$100,000 in an account. How much money was invested in this account 10 years ago to earn this amount? (No money was added in the mean time.)

70. A rectangle with sides parallel to the axis is to have one corner at the origin and the other on the curve $y = e^{-x^2}$. What is the largest rectangular area that can be formed in this way?

71. In a solution, the product of the hydronium ion concentration $[\text{H}_3\text{O}^+]$ and the hydroxyl ion concentration $[\text{OH}^-]$ (both in moles per liter) is 10^{-14} .

- What value of $[\text{H}_3\text{O}^+]$ minimizes the sum $S = [\text{OH}^-] + [\text{H}_3\text{O}^+]$?
- What is the pH of solution you found in part a)?
- What ratio of $[\text{H}_3\text{O}^+]$ to $[\text{OH}^-]$ minimizes S ?

72. Compute

$$\sum_{n=0}^{\infty} \frac{\pi^n}{4^n}$$

73. Compute

$$\sum_{n=0}^{\infty} (-1)^n \frac{1}{5^n}$$

74. Compute

$$\sum_{n=1}^{\infty} \frac{4}{(4n-3)(4n+1)}$$

75. Determine if the series converges or diverges. Justify carefully.

$$\sum_{n=1}^{\infty} 1.000001^n$$

76. Compute and state for which values of x the series converges

$$\sum_{n=0}^{\infty} e^{nx}$$

77. Compute

$$\sum_{n=1}^{\infty} \frac{1}{4n^2 - 1}$$

78. Compute write the number $2.373737\dots$ as a ratio of two integers.

79. Assume that $|x| < 1$ and compute the sum

$$\sum_{n=0}^{\infty} x^n.$$

Use your answer to compute the sum

$$\sum_{n=1}^{\infty} nx^{n-1}.$$