

Compound Interest

In this handout, we will use exponential and logarithmic functions to answer questions about interest earned on investments (or charged when money is borrowed).

SIMPLE INTEREST If a principal P is borrowed for a period of t years at a per annum interest rate r , expressed as a decimal, the interest I charged is

$$I = Prt$$

Compound Interest: interest is earned (or charged) on a regular schedule (e.g. once a year, every month, etc.); at the end of each **payment period**, interest is earned on principal and on previously earned interest

Example. Gertrude invests \$300 in a savings plan that earns 11% per annum compounded quarterly. How much will be in Gertrude's account after one year?

	principal + interest
initial investment	
end of 1st quarter	
end of 2nd quarter	
end of 3rd quarter	
end of 4th quarter	

COMPOUND INTEREST

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$

A = amount in account after t years

P = amount invested (or borrowed)

r = per annum interest rate (expressed as a decimal)

n = number of times interest is compounded per year

t = time (in years)

Problem 1. Suppose \$300 is invested in an account that pays 11% per annum.

- (a) If interest is compounded quarterly, how much money will be in the account after 2 years? (This amount is called **future value**.)

- (b) What happens to the amount in the account after 2 years as you compound more and more times per year?

payment period	# times compounded per yr.	\approx amount after 2 yrs.
annually		
semiannually		
quarterly		
monthly		
daily		
continuously		

CONTINUOUSLY COMPOUNDED INTEREST The amount A after t years due to a principal P invested at an annual interest rate r compounded continuously is

$$A = Pe^{rt}$$

Problem 2. A savings plan offers a rate of 8% compounded quarterly. How much should be invested now in order to have \$1000 after 5 years? (This amount is called **present value**.)

Problem 3. Suppose \$500 are invested at 9% per annum. If interest is compounded continuously, how long will it take for \$500 to double to \$1000?

Problem 4. If interest is compounded annually, what interest rate should you seek if you want to triple your investment in 10 years?

Problem 5. Consider the following two investments:

Option 1 invest \$1000 at a rate of 5%, compounded quarterly

Option 2 invest \$1000 at rate R , compounded annually

What rate R should you seek in order to have the same amount in each account after 1 year? (This rate is called the **effective interest rate** or the **annual percentage yield**.)