Math 1710.007, Spring 2012
Review for Exam 3

The third exam is **Wednesday, April 18**. We will have lecture from 6-6:50pm, and the exam will be 7:00-7:50pm. The exam will cover material from Chapter 4 (Sections 4.4,4.6-4.8), Chapter 5 (Sections 5.1-5.4), and Chapter 6 (Section 6.1). Calculators are **NOT** permitted on the exam. Good review: problems on this sheet, homework and quiz problems, examples from class, and worked examples from the textbook. The actual exam will be mostly free response questions (i.e. "show your work" problems), but there may also be some multiple choice, matching, or fill in the blank questions. The test may include problems that do not look exactly like the ones on this sheet!

1. (4.4) #10b, 12, 15, 16, 31, 37 from p.213-216 of the textbook.

2. (4.4) Find the point on the graph of $y = \sqrt{x}$ that is closest to the point $(5, 0)$. (*Hint:* You can minimize the square of the distance.)

3. (4.6) What is the average rate of change of $f(x) = \frac{x}{x+2}$ on the interval $[1, 4]$? At what x-values does the instantaneous rate of change equal the average rate of change?

4. (4.6) Usain Bolt set a world record of 9.58 s in the 100-meter dash in the summer of 2009. Did his speed ever exceed 37 km/hr during the race? Explain.

5. (4.7) Evaluate the limits.

   - $\lim_{\theta \to \frac{\pi}{4}} \dfrac{\tan\theta - \cot\theta}{\theta - \frac{\pi}{4}}$
   - $\lim_{x \to 2} \dfrac{x^3 - 3x^2 + 4}{x^4 - 4x^3 + 7x^2 - 12x + 12}$
   - $\lim_{x \to 0} \dfrac{2\sin x - \sin(2x)}{\sin x - x\cos x}$

6. (4.8) If the velocity function is

   $$v(t) = \sin t + 3\cos t,$$

   and $s(0) = 4$, find the position function $s(t)$.

7. (4.8) #17-30 on p.247 of the textbook (indefinite integrals)

8. (1.3) Review the unit circle, key trig values, and trig identities (especially Reciprocal, Pythagorean, Double-Angle, Half-angle)

9. (4.8) Find the integrals below. (First use a trig identity to rewrite the integrand so you get something easier to work with.)

   - $\int \sin^2 t \, dt$
   - $\int \cos^2 t \, dt$
   - $\int \sin t \cos t \, dt$
   - $\int \tan^2 t \, dt$

10. (5.1) #57 on p.264 of the textbook (approximating area from a graph)

11. (5.1) #16 on p.261 of the textbook (left and right Riemann sums)

12. (5.1) Suppose a robot travels at velocity

    $$v(t) = \sqrt{7t},$$

    in meters per second. Approximate the robot's displacement during the time interval $[3, 8]$:

    (a) using a left Riemann sum with $n = 10$ rectangles.

    (b) using a right Riemann sum with $n = 10$ rectangles.

    Your work for each part should include a table listing the time intervals, sample times, sample velocities, change in time, and approximate displacements.

13. (5.1) Suppose a robot travels at velocity

    $$v(t) = \sqrt{7t},$$

    in meters per second. The displacement over the time interval $[0, 5]$ can be approximated using a left Riemann sum with $n = 50$ rectangles. Use sigma notation to write the Riemann sum. (You don't have to evaluate it; just set it up.)

14. (6.1) #41 on p.324 of the textbook (velocity graphs)

15. (6.1) Two cars start at the same place and drive in the same direction along a straight highway for 1 hour. In miles per hour, Car A drives at velocity $v_A(t) = 60\sqrt{t}$, and Car B drives at velocity $v_B(t) = 60t^2$.

    (a) Sketch the velocity graphs together on the same axes.

    (b) Which car is going faster after 45 minutes?

    (c) What is the distance between the cars after 1 hour?

16. (5.2) #41 on p.277 of the textbook (properties of integrals)

17. (5.2) Evaluate the definite integrals using geometry. Sketch a graph of the integrand and shade the region whose net area you have found.

    - $\int_{-8}^{4} (|x+2| - 3)\, dx$
    - $\int_{-5}^{5} \sqrt{25 - x^2}\, dx$
    - $\int_{0}^{\sqrt{2}/2} \sqrt{1 - x^2}\, dx$

18. (5.3) #25-37 odd on p.291-292 of the textbook (definite integrals)

19. (5.2) #31-34 on p.277 of the textbook (net area from graphs)

20. (5.2) Let $v(t) = 14 - 3t$ be the velocity function. The displacement/net area function based at $t = 1$ is
    $$A_1(x) = \int_1^x v(t)dt$$
    Sketch the velocity function and use geometry to find $A_1(1)$ and $A_1(3)$.

21. (5.3) Simplify $\dfrac{d}{dx} \displaystyle\int_0^x \frac{3t^7 - \sin t}{\sqrt{t^2 + 16}}\, dt$.

22. (5.4) Use symmetry to help evaluate the definite integrals.

    - $\int_{-\pi}^{\pi} \sin(2\theta)\, d\theta$
    - $\int_{-\pi/2}^{\pi/2} \cos\theta\, d\theta$
    - $\int_{-1}^{1} (4x^5 - 5x^4 + 3x - 15)\, dx$

23. (5.4) If you travel at velocity $v(t) = 1 - \frac{1}{16}t^2$ for $0 \le t \le 4$, what is your displacement? At what constant velocity would you have to travel to achieve the same displacement? Illustrate on a graph.