Math 1710.007, Spring 2012
Review for Exam 1

The first exam is **Wednesday, February 15**. We will have lecture from 6-6:50pm, and the exam will be 7:00-7:50pm. The exam will cover Chapter 2 (Sections 2.1-2.7) and Chapter 3 (Section 3.1). You may also need to review Chapter 1 material, especially the graphs of basic functions, trig values/unit circle, and trig identities. Calculators are **NOT** permitted on the exam. Good review: problems on this sheet, homework and quiz problems, examples from class, and worked examples from the textbook. The actual exam will be mostly free response questions (i.e. "show your work" problems), but there may also be some multiple choice, matching, or fill in the blank questions. The test may include problems that do not look exactly like the ones on this sheet!

1. (1.3) Review trigonometry: unit circle and key values of trig functions, reciprocal identities, Pythagorean identities

2. (2.1) Suppose the position of an object moving along a straight line is given by the function $s(t) = -16t^2 + 128t$. Fill in the blanks: At time $t = 1$, the position is $s(1) =$ ____. Two seconds later, the time is $t =$ ____, and the position is $s($____$) =$ ____. The average velocity over the interval $[1, 3]$ is ____. If instead you wait h seconds before measuring the position again, the time will be $t = 1 + h$, and the position will be $s(1 + h) =$ ____. The average velocity over the interval $[1, 1 + h]$ is ____. When you take the measurements closer together, i.e. let h approach zero, the average velocity v_{av} approaches the instantaneous velocity $v_{inst} =$ ____.

3. (2.2) #2 on p.97 of the textbook (estimating limits graphically)

4. (2.3) Evaluate the limits.
   - $\displaystyle\lim_{h\to 0} \frac{(7+h)^2 - 49}{h}$
   - $\displaystyle\lim_{h\to 0} \frac{\frac{1}{6+h} - \frac{1}{6}}{h}$
   - $\displaystyle\lim_{x\to -2} \frac{x^2 - 4}{x^2 + 7x + 10}$
   - $\displaystyle\lim_{\theta\to \pi} \frac{\tan\theta}{\sin\theta}$

5. (2.3) Evaluate the limits.
   - $\displaystyle\lim_{h\to 0} \frac{\sqrt{64+h} - 8}{h}$
   - $\displaystyle\lim_{\theta\to 0} \frac{1 - \cos\theta}{\sin^2\theta}$

6. (2.3) The goal of this problem is to evaluate $\displaystyle\lim_{x\to 0} x^4 \sin\left(\frac{1}{x}\right)$.

   (a) Fill in the blanks: The maximum value of $\sin\theta$ is ____. The minimum value of $\sin\theta$ is ____. So

   $$\_\_\_\_ \le \sin\left(\frac{1}{x}\right) \le \_\_\_\_.$$

   Multiply all sides of the inequality by x^4:

   $$\_\_\_\_ \le x^4 \sin\left(\frac{1}{x}\right) \le \_\_\_\_$$

   (The inequality signs stay in the same direction since x^4 is never negative.)

   (b) Use the Squeeze Theorem to evaluate $\displaystyle\lim_{x\to 0} x^4 \sin\left(\frac{1}{x}\right)$.

7. (2.3) Suppose $2|x - \pi| \le f(x) \le -1 - \cos(x)$ for all real numbers x. Find $\displaystyle\lim_{x\to\pi} f(x)$.

8. (2.4) Evaluate the limits.
   - $\displaystyle\lim_{x\to -1^-} \frac{1}{(x+1)^4}$ and $\displaystyle\lim_{x\to -1^+} \frac{1}{(x+1)^4}$
   - $\displaystyle\lim_{x\to 3^-} \frac{1}{(x-3)^5}$ and $\displaystyle\lim_{x\to 3^+} \frac{1}{(x-3)^5}$

9. (2.4) Evaluate the limits. Your work should include a sign chart.
   - $\displaystyle\lim_{x\to 6^-} \frac{x^2+1}{x^2 - 5x - 6}$ and $\displaystyle\lim_{x\to 6^+} \frac{x^2+1}{x^2 - 5x - 6}$
   - $\displaystyle\lim_{\theta\to \pi^-} \cot\theta$ and $\displaystyle\lim_{\theta\to \pi^+} \cot\theta$
   - $\displaystyle\lim_{x\to -2^-} \frac{\cos x}{x^2 + 2x}$ and $\displaystyle\lim_{x\to -2^+} \frac{\cos x}{x^2 + 2x}$

10. (1.2) Sketch the basic shape of the graphs of $y = x^3, x^5, x^7, \ldots$ and of $y = x^2, x^4, x^6, \ldots$.

11. (1.2) Sketch the basic shape of the graphs of $y = \frac{1}{x}, \frac{1}{x^3}, \frac{1}{x^5}, \ldots$ and of $y = \frac{1}{x^2}, \frac{1}{x^4}, \frac{1}{x^6}, \ldots$.

12. (2.5) Determine the end behavior for the polynomial $p(x) = -3x^{10} + 8x^3 - 4x^5 + x - 9$ by evaluating the limits $\displaystyle\lim_{x\to -\infty} p(x)$ and $\displaystyle\lim_{x\to\infty} p(x)$.

13. (2.5) Determine the end behavior for each function by evaluating the limits $\displaystyle\lim_{x\to -\infty} f(x)$ and $\displaystyle\lim_{x\to\infty} f(x)$ and then giving the horizontal asymptote(s) of f, if any.

- $f(x) = \dfrac{3x^4 - 10x^2 + x - 6}{5x^4 - 13x + 11}$
- $f(x) = \dfrac{2x^2 - 2x + 1}{6x^3 - 3x^2}$
- $f(x) = \dfrac{4x^3 + 1}{2x^3 + \sqrt{16x^6 + 5x^4 + 1}}$

14. (2.5) Let $r(x) = \dfrac{x^4 - 2x^3 - 2x^2 + 6x - 1}{x^2 - 2x + 1}$.

    (a) Use long division to find the polynomial $Q(x)$ that the graph of r will resemble for large values of x.

    (b) Use a graphing calculator to see how the graph of the rational function $r(x)$ compares with the graph of the polynomial $Q(x)$. (On the test you won't be asked to do this part.)

    (c) Evaluate $\lim_{x\to-\infty} r(x)$ and $\lim_{x\to\infty} r(x)$.

15. (2.5) The goal of this problem is to evaluate $\lim_{x\to\infty} \dfrac{\sin x + 5}{x}$.

    (a) Fill in the blanks: The maximum value of $\sin x$ is ____. The minimum value of $\sin x$ is ____. So

    $$\_\_\_\_ \le \sin x \le \_\_\_\_.$$

    Then

    $$\_\_\_\_ \le \sin x + 5 \le \_\_\_\_,$$

    and if x is positive,

    $$\_\_\_\_ \le \frac{\sin x + 5}{x} \le \_\_\_\_.$$

    (b) Use the Squeeze Theorem to evaluate $\lim_{x\to\infty} \dfrac{\sin x + 5}{x}$.

16. (2.6) #3 on p.97 of the textbook (finding points of discontinuity graphically)

17. (2.6) Determine the points of discontinuity, and classify the discontinuities as jump, removable, infinite, or oscillating.

    - $f(x) = \dfrac{|x|}{x}$
    - $g(x) = \sin(\frac{1}{x})$
    - $h(x) = \lfloor x \rfloor$ (floor function)
    - $R(x) = \dfrac{x^3 - 4x^2 + 4x}{x(x-1)}$

18. (2.6) For what value of a is the following function continuous?

    $$f(x) = \begin{cases} a - 3x^2 & x \le 2 \\ 5x - 8 & x > 2 \end{cases}$$

19. (2.6) Show the equation $x^4 + 3x - 1 = 0$ has at least one solution between -1 and 1. Justify your answer carefully.

20. (2.7) #11 on p.93 of the textbook (finding values of δ from a graph)

21. (2.7) The function $F(x) = \frac{9}{5}x + 32$ converts degrees Celcius to degrees Fahrenheit. Note that $\lim_{x\to 25} F(x) = 77$.

    (a) ($\epsilon = 0.1$) Find a number $\delta > 0$ such that: if

    $$0 < |x - 25| < \delta,$$

    then

    $$|F(x) - 77| < 0.1$$

    What is the biggest choice of δ that will work? Which of the δ values 1, .1, .01, .001, .0001 would also work? Illustrate on a graph.

    (b) Find a formula for δ (in terms of ϵ) such that: if

    $$0 < |x - 25| < \delta,$$

    then

    $$|F(x) - 77| < \epsilon.$$

22. (3.1) Find the derivative of each function using the limit definition of the derivative.

    - $f(x) = x^2$
    - $f(x) = x^3$
    - $f(x) = \dfrac{1}{x}$
    - $f(x) = \sqrt{x}$
    - $f(x) = \frac{1}{\sqrt{x}}$

23. (3.1) Find an equation for the line tangent to the graph of $f(x) = x^2$ at the point $(3, 9)$. Write the equation of the line in the form

    $$y = m(x - a) + f(a)$$

24. (3.1) Draw an example of a function that is continuous at $x = 0$ but $f'(0)$ is undefined.

25. (3.1) #39 on p. 110 of the textbook (derivatives from graphs)

26. (3.1) Go to http://rowdy.mscd.edu/~talmanl/MTH1410U08/Pictures_080529 to practice matching functions with their derivatives.