MATH 1710 Review sheet for Final Exam

contributed by

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Chapter 2 Sample Problems

1. Compute the following limits:

a) $\displaystyle\lim_{x\to 1} \frac{x+1-\sqrt{x+3}}{x-1}$

b) $\displaystyle\lim_{x\to\infty} \frac{x^2+3x-1}{2x-5x^2}$

c) $\displaystyle\lim_{x\to 2} \frac{x^2-7x+6}{x+2}$

d) $\displaystyle\lim_{z\to 6^-} (3z^{-1}+\sqrt{22-z})$

e) $\displaystyle\lim_{t\to -5} \frac{t^2+3t-10}{t+5}$

f) $\displaystyle\lim_{x\to 7^-} \frac{10}{x-7}$

g) $\displaystyle\lim_{\theta\to -\frac{\pi}{2}^+} \tan\theta$

h) $\displaystyle\lim_{x\to -\infty} (-\frac{2}{x^2}+11)$

i) $\displaystyle\lim_{x\to\infty} \frac{x^2+1}{100x}$

j) $\displaystyle\lim_{x\to\infty} \frac{8x^4-5x+18}{3x^4-3}$

k) $\displaystyle\lim_{\theta\to\infty} \frac{10\sin\theta}{\theta}$

2. Suppose $2|x-\pi| \leq f(x) \leq -1-\cos x$ for all real numbers x. Find $\lim_{x\to\pi} f(x)$.

3. Determine whether the following function is continuous at $x=3$, and explain why or why not.

$$f(x) = \begin{cases} \dfrac{x^2-9}{x-3} & : \quad x \neq 3 \\ 0 & : \quad x = 3 \end{cases}$$

4. For what value of a is the following function continuous?

$$f(x) = \begin{cases} a - 3x^2 & : \; x \leq 2 \\ \\ 5x - 8 & : \; x > 2 \end{cases}$$

5. Use the Squeeze Theorem to evaluate the limit

$$\lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right)$$

6. Find the vertical asymptotes of the function

$$f(x) = \frac{x^2 - 4x + 3}{x^2 + x - 2}$$

7. Evaluate the limit

$$\lim_{x \to 1^+} \frac{-1}{x^2 - 1}$$

(a) $+\infty$

(b) $-\infty$

(c) 0

(d) does not exist

8. Evaluate the limit

$$\lim_{\theta \to 0^-} \cot(\theta)$$

9. Identify any horizontal asymptotes for the function

$$f(x) = \frac{2x^3 - 10x + 7}{-6x^3 + x^2 - 12}$$

10. Find the limit

$$\lim_{x \to -\infty} \frac{5 + \sin(x)}{x^2}$$

(a) 0

(b) $-\infty$

(c) 5

(d) does not exist

11. Find the limit

$$\lim_{x\to\infty} \frac{-x^4+5x^3-2x+9}{2x^3+2x-3}$$

12. Explain why the following function is continuous at $x=3$

$$f(x) = \begin{cases} \frac{x^2-9}{x-3} & \text{if } x \neq 3 \\ 6 & \text{if } x = 3 \end{cases}$$

13. Find the limit

$$\lim_{x\to 0^+} \frac{-5x}{|x|}$$

(a) 5

(b) -5

(c) -1

(d) does not exist

14. Suppose $\lim_{x\to 3} f(x) = -2$ and $\lim_{x\to 3} g(x) = 5$. Find the limit

$$\lim_{x\to 3}(3f(x)+2g(x))^{-1/2}.$$

15. Find the limit

$$\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2}$$

(a) 4

(b) 1

(c) 0

(d) undefined

16. Find the limit

$$\lim_{x\to 1} \frac{x^2+2x-3}{x-1}$$

17. Show that the equation $x^4+3x-1=0$ has at least one solution on $(-1,1)$. Justify your answer carefully.

Chapter 3 Sample Problems

1. If a function f is continuous, then

(a) f is differentiable

(b) f' is continuous

(c) f' is differentiable

(d) none of above

2. Where does the function $y = sin(x)$ have a horizontal tangent line?

(a) 0

(b) 1

(c) $\pi/2$

(d) $\pi/6$

3. Find dy/dx for the equation $xy + x + y = x^2y^2$.

4. Find dy/dx for $y = (2x+1)^2(3x^2+2)$.

5. Find dy/dx for $y = \frac{x^3-4x^2+x}{x-2}$.

6. Find d^2y/dx^2 for $y = sec(x)$.

7. Flying a kite. Once Kate's kite reaches a height of 50 ft (above her hands), it rises no higher but drifts due east in a wind blowing at 5 ft/s. How fast is the string running through Kate's hands at the moment that she has released 120 ft of string?

8. For the curve $x^2 - xy + y^2 = 12$,

a) Show that

$$\frac{dy}{dx} = \frac{2x-y}{x-2y}$$

b) Find the point or points where the curve has a horizontal tangent line.

9. Sand falls from a conveyor belt at the rate of 10 m^3/min onto the top of a conical pile. The height of the pile is always three-eights of the base diameter. How fast are the **(a)** height, and **(b)** radius changing when the pile is 6 m high? (You don't need to write your answers as decimal approximations.)

10. Differentiate the following functions:

a) $f(x) = 5\cos(3x)$

b) $g(x) = \sqrt{\sin^2 x}$

11. Find an equation for the line tangent to the elipse $x^2 + 3y^2 = 28$ at the point $(4, 2)$.

12. A police car is pursuing a suspect who is fleeing west in their car down Main Street. At $t = 0$, the suspect passes through the intersection of Main and Pine, traveling west at 80 ft/sec. At the same time, the police car is 300 ft. south of the intersection, going north on Pine toweards Main at 100 ft/sec. Assume that both cars maintain a constant speed. Is the police car getting closer to the suspect or further when it is 200 feet south of the intersection? How quickly is the distance between tem changing?

13. Find the slope of the curve $y = \dfrac{1}{x}$ at the point $(2, 1/2)$.

14. Use *the definition of a derivative* (NOT the power rule!) to prove that if $y = x^3$, then $dy/dx = 3x^2$.

15. Calculate the derivatives using a method of your choice. Simplify your answers!

a) $y = \sqrt{x} - 7x^{-1/3} + \pi$

b) $y = (x^2 + 1)(x^3 - 2)$

c) $y = \dfrac{\sin x}{\tan x}$

16. Find the x-coordinates of all points where the curve $y = x^3 - 6x^2 - 15x + 1$ has a horizontal tangent line. (You need not compute the y-coordinates). Sketch the curve and the horizontal tangent line together.

17. If f is continuous at $x = 0$, does f always have a derivative at $x = 0$? Give an example to support your answer.

Chapter 4 Sample Problems

1. Find the absolute minimum and maximum of $f(x) = \sqrt{1 + x^2} - 2x$ on the interval $[0, 1]$.

2. Use Rolle's Theorem to explain why there is no $x > 0$ which is a root of $x^4 + 5x^3 + 4x$ (Note $x = 0$ is a root).

3. Suppose that f is a function so that $f'(x) = (x + 3)^2(2 - x)^3(x - 4)$. Which of the following is true?

I.) f has a local minimum at $x = 2$
II.) f has a local minimum at $x = -3$
III.) f has a local maximum at $x = 4$

a.) I. only
b.) II. only
c.) I. and II.
d.) I. and III.
e.) None of the above.

4. Find a value of c which satisfies the conclusion of the Mean Value Theorem for $f(x) = \dfrac{x}{x+2}$ on $[1, 4]$.

5. On which of the following intervals is $f(x) = x^2 + bx + c$ increasing?

a.) $(-\infty, \infty)$
b.) $(-\infty, \frac{b}{2})$
c.) $(-\frac{b}{2}, \infty)$
d.) $(-\frac{b}{2}, \frac{b}{2})$

6. Consider the following function $f(x) = x - \dfrac{48}{x^2}$.
a.) Give the domain of f.
b.) Determine the intervals where f is increasing and decreasing.
c.) Find any local extreme values.
d.) Describe where f is concave up and down.
e.) Find any inflection points.
f.) Sketch a graph of f labelling all relevant points.

7. Postal regulations require that the sum of the height and girth (perimeter of it's base) of a box not exceed 108 in^3. If the box is to have a square base, find the maximum volume possible.

8. An architect wishes to enclose a rectangular garden of area $1,000$ m^3 on one side by a brick wall costing \$90 per meter, and on the other 3 sides by a metal fence costing \$30 per meter. Which dimensions minimize the total cost?

9. Find (if possible) the coordinates of the point on the graph of $y = x + \dfrac{2}{x}$ closest to the origin inside the region $x > 0$.

10. Evaluate the limit (if it exists): $\displaystyle\lim_{x\to 0} \frac{2\sin x - \sin 2x}{\sin x - x\cos x}$.

11. Determine the constants a, and b which force the function $f(x) = x^2 + ax + b$ to have a relative minimum at $x = 2$ and so that $f(2) = 7$.

12. Find the inflection points of $f(x) = \sin 2x - 4\cos x$.

13. Consider $\lim_{x \to 0^+} \dfrac{\sqrt{x}}{\sin x}$

a.) ∞
b.) does not exist
c.) 0
d.) 1

14. Suppose that $\dfrac{dy}{dt} = 3t^2 + \cos t$, and $y(0) = 12$. Find $y(t)$.

15. Let $f(x) = x^{7/3} - 7x^{4/3}$.

a) Find the intervals where f is increasing and decreasing, and find all local maxima and minima.

b) Find the intervals where the graph of f is concave up and concave down, and find all inflection points (if any).

c) Sketch a graph of f, and LABEL all intercepts, local extrema, and inflection points.

16. Which of the following integration formulas is correct? Give reasons for your answer! (It is not necessary to explain why the others are wrong).

a) $\displaystyle\int x\cos 2x\,dx = \frac{x^2}{2}\sin 2x + C$

b) $\displaystyle\int x\cos 2x\,dx = \frac{x}{2}\sin 2x + \frac{1}{4}\cos 2x + C$

c) $\displaystyle\int x\cos 2x\,dx = x\sin 2x + \cos 2x + C$

17. Suppose that the revenue from selling x items is determined by $r(x) = 9x$ and $c(x) = x^3 - 6x^2 + 15x$ is the cost of manufacturing x items. Is there a production level that maximizes profit? If so, what is it?

Chapter 5 Sample Problems

1. Evaluate the following integrals:

a) $\displaystyle\int_0^{\pi/4} \sec^2 x \tan x\,dx$

b) $\displaystyle\int \frac{x^2}{(x^3+1)^2}\,dx$

c) $\int_{-4}^{4} \sqrt{16-x^2}\,dx$

d) $\int_{-2}^{2} (3x^4 - 2x + 1)\,dx$

e) $\int \cos 3x\,dx$

f) $\int \frac{x}{\sqrt{x-4}}\,dx$

g) $\int_{0}^{4} \frac{p}{\sqrt{9+p^2}}\,dp$

2. Assume f' is a continuous function, $\int_1^2 f'(2x)\,dx = 10$, and $f(2) = 4$. Evaluate $f(4)$.

3. Suppose that $\int_1^4 f(x)\,dx = 6$, $\int_1^4 g(x)\,dx = 4$, and $\int_3^4 f(x)\,dx = 2$. Evaluate the following integral $\int_3^1 (f(x) - g(x))\,dx$, or state that there is not enough information.

4. Consider the integral $\int_1^4 (3x-2)\,dx$.

a) Evaluate the right Riemann sum for the integral with in $n = 3$.

b) Use summation notation to write the right Riemann sum of an arbitrary positive integer n.

c) Evaluate the definite integral by taking the limit as $n \to \infty$ of the Riemann sum in part (b).

5. An object travels on the x-axis with a velocity given by $v(t) = 2t+5$, for $0 \le t \le 4$.

a) How far does the object travel, for $0 \le t \le 4$.

b) What is the average of v on the interval $[0, 4]$.

c) True or False: The object wouls travel as far as in part (a) if it traveled at its average velocity (a constant), for $0 \le t \le 4$.

6. Suppose you walk at a constant speed along a semicircle with a raduis of $1km$. What is your average distance from the base of the semicircle over your entire trip?

7. State the Fundamental theorem of calculus.

8. Consider the trapezoid bounded by the line $y = 2t + 3$ and the x-axis from $t = 2$ to $t = x$. The Area function $A(x) = \int_2^x f(t)\,dt$ gives the area of the trapezoid, for

$x \geq 2$.
a) Evaluate $A(2)$.
b) Evaluate $A(5)$.
c) Find and gragh the function $y = A(x)$, for $x \geq 2$.
d) Compute the derivative of A to f.

9. You are given that $\int_0^6 f(x)\,dx = 18$, and that $\int_0^1 \{1 + f(x)\}\,dx = 8$. Find $\int_1^6 f(x)\,dx$.

10. Suppose the interval $[1, 3]$ is partitioned into $n = 4$ subintervals. What is the subinterval length $\triangle x$? List the grid points x_0, x_1, x_2, x_3, x_4. What points are used for the left, right, and midpoint Riemann sums?

11. Evaluate the following expression: $\Sigma_{p=1}^{5}(2p + p^2)$.

12. Draw a graph of each integrand, and then use areas to calculate the integrals.

a)

$$\int_{-2}^{2} (2 - |x|)\,dx$$

b)

$$\int_{0}^{\sqrt{2}/2} \sqrt{1 - x^2}\,dx$$

(*Hint*: This is the area of a circle sector plus a triangle!)

13. For the function $f(x) = \sin 4x$:

a) Find the area between the x-axis and the graph of f between $x = 0$ and $x = \pi/4$.

b) Find the average value of $f(x)$ on the interval $[0, \pi/4]$. (Hint: how is the related to the answer to part (a)?)

14. Find or approximate the point(s) at which $f(x) = 1/x^2$ equals its averate value on the interval $[1, 4]$.

15. Express the following as a definite integral.

$$\lim_{\|P\| \to 0} \sum_{k=1}^{n} (x_k^3 + 4x_k - 1)\Delta x_k,$$

where P is a partition of $[0, 7]$, and $x_k \in \Delta x_k$.

16. Evaluate the following derivatives, and simplify as needed.

a) $\frac{d}{dx}\int_2^x \sin(\sqrt{t})\,dt =$

b) $\frac{d}{dx}\int_{\sqrt{x}}^3 \sqrt{t^2+1}\,dt$

17. Define the anti-derivative of a function f.

18. Definie the Riemann Integral of a function f over the interval $[a,b]$.

19. Let $y = \int_2^{x^3} \sin(3t+\sqrt{t})\,dt$. Find dy/dx.

Chapter 6 Sample Problems

1. Find the area of the region enclosed by the curves $y = x^3 - x^2$ and $y = x^3 + x - 2$.

2. Find the area of the region enclosed by the curves $y = x + 3$ and $y = |2x|$.

3. Find the area of the region enclosed by the curves $y = \sin x$ and $y = \cos x$, and x-axis between $x = 0$ and $x = \pi/2$.

4. Find the area of the region enclosed by the curves $y = 2x$, and $y = 1 - 2x^2$.

5. Find the area of the region enclosed by the curves $x = y^2 - 4$ and $y = x/3$ by integrating with respect to y.

6. The region between the curve $y = \sqrt{x} + 1, 0 \le x \le 4$, and the x-axis is revolved about the x-axis to generate a solid. Find its volume. Show all your logic and work clearly.

7. Find the volume of the solid generated by revolving the region between the curve $y = \sin x$, the line $y = 1$, the line $x = 0$ and the line $x = 2\pi$ about the line $y = 1$.

8. Find the volume of the solid generated by revolving the region enclosed by the curve $y = 3 - x^2$ and the line $y = 2$ about the x-axis.

9. Let R be the region bounded by the curve $y = 2\sqrt{x}$ and $y = x$. Use the washer method to find the volume of the solid generated when R is revolved about the x-axis.

10. Let R be the region bounded by the curve $y = 3x, x = 0$ and $y = 3$. Use the shell method to find the volume of the solid generated when R is revolved about the y-axis.

11. Find the length of the curve $f(x) = x^{\frac{3}{2}}$ on the interval $[0, 4]$.

12. Consider the segment of $f(x) = x^2 - 7$ on the interval $[0, 3]$. Write the integral for the length of the curve.

13. How much work is required to move an object from $x = 0$ and $x = 5$ (measured in meters) in the presence of a constant force of $5N$ acting along the x-axis?