MATH 1710 Answer for Review sheet

contributed by

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Chapter 2 Sample Problems

1.
   a) $3/4$ b) $-1/5$ c) -1 d) $9/2$ e) -7 f) $-\infty$ g) $-\infty$

   h) 11 i) ∞ j) $\frac{8}{3}$ k) 0

2. 0

3. No, because $\lim_{x\to 3} f(x) = 6$ but $f(3) = 0$.

4. $a = 14$

5. 0

6. $f(x)$ has a vertical asymptotes at $x = -2$

7. b

8. $-\infty$

9. $f(x)$ has a horizontal asymptote at $y = -\frac{1}{3}$.

10. a

11. $-\infty$

12. $\lim_{x\to 3} f(x) = \lim_{x\to 3} \frac{x^2-9}{x-3} = 6 = f(3)$

13. b

14. $\frac{1}{2}$

15. a

16. 4

17. Let $f(x) = x^4 + 3x - 1$. Since $f(-1) < 0$ and $f(1) > 0$, the Intermediate Value Theorem implies that there must exist at least one $c \in (-1, 1)$ such that $f(c) = 0$.

Chapter 3 Sample Problems

1. (D)

2. (C)

3. Differentiate both sides with respect to x,
$y + x\frac{dy}{dx} + 1 + \frac{dy}{dx} = 2xy^2 + 2x^2y\frac{dy}{dx}$
$(-2x^2y + x + 1)\frac{dy}{dx} = 2xy^2 - y - 1$
$\frac{dy}{dx} = \frac{2xy^2-y-1}{-2x^2y+x+1}$

4. By product rule, $dy/dx = [\frac{d}{dx}(2x+1)^2](3x^2+2) + (2x+1)^2[\frac{d}{dx}(3x^2+2)]$
$= 2(2x+1)[\frac{d}{dx}(2x+1)](3x^2+2) + (2x+1)^2[6x]$
$= (8x+4)(3x^2+2) + (2x+1)^2(6x)$
$= (24x^3 + 12x^2 + 16x + 8) + (4x^2 + 4x + 1)(6x)$
$= (24x^3 + 12x^2 + 16x + 8) + (24x^3 + 24x^2 + 6x)$
$= 48x^3 + 36x^2 + 22x + 8$

5. By quotient rule, $dy/dx = \frac{[\frac{d}{dx}(x^3-4x^2+x)](x-2)-(x^3-4x^2+x)[\frac{d}{dx}(x-2)]}{(x-2)^2}$
$= \frac{(3x^2-8x+1)(x-2)-(x^3-4x^2+x)}{(x-2)^2}$
$= \frac{(3x^3-14x^2+17x-2)-(x^3-4x^2+x)}{(x-2)^2}$
$= \frac{2x^3-10x^2+16x-2}{(x-2)^2}$

6. $dy/dx = sec(x)tan(x)$, so $d^2y/dx^2 = \frac{d}{dx}[\frac{dy}{dx}]$
$= [\frac{d}{dx}sec(x)]tan(x) + sec(x)[\frac{d}{dx}tan(x)]$
$= sec(x)tan(x)tan(x) + sec(x)sec^2(x)$
$= sec(x)tan^2(x) + sec^3(x)$

7. Analysis: Kate's hand, the point 50ft above Kate's hand and the position of the kite always form a rectangular triangle. Let the hypotenuse be s, two sides be x and y. According to the problem, $y = 50ft$, $\frac{dx}{dt} = 5ft/s$ and $s = 120ft$ at that time. We need to find $\frac{ds}{dt}$.

First, by Pythagorean theorem, $s^2 = x^2+y^2$. Hence at that time $x^2 = 120^2-50^2 = 11900$, $x \approx 109.09ft$.

Second, differentiate both sides of Pythagorean theorem with respect to t, we have $2s\frac{ds}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$.

At that time $y = 50ft$, $\frac{dy}{dt} = 0$(why?), $x \approx 109.09ft$, $\frac{dx}{dt} = 5ft/s$, $s = 120ft$.
So we get $2(120)(\frac{ds}{dt}) = 2(109.09)(5)$
$\frac{ds}{dt} \approx 4.55ft/s$.

8. (a) $x^2 - xy + y^2 = 12 \Rightarrow 2x - (xy' + 1 \cdot y) + 2y \cdot y' = 0 \Rightarrow 2yy' - xy' = y - 2x \Rightarrow$
$y'(2y - x) = y - 2x \Rightarrow y' = \frac{y-2x}{2y-x} = \frac{2x-y}{x-2y}$

(b) $y' = 0 \Rightarrow \frac{2x-y}{x-2y} = 0 \Rightarrow 2x - y = 0,\ x - 2y \neq 0$
$\Rightarrow y = 2x,\ x \neq 2y \Rightarrow y = 2x,\ (x,y) \neq (0,0) \Rightarrow y = 2x$ and $x^2 - xy + y^2 = 12 \Rightarrow$
$x^2 - x(2x) + (2x)^2 = 12 \Rightarrow x^2 - 2x^2 + 4x^2 = 12 \Rightarrow 3x^2 = 12 \Rightarrow x = \pm 2$
Therefore, we have horizontal tangents at $(2, 4)$ and at $(-2, -4)$

9. $h = \frac{3}{8}\,(\text{diam.}) = \frac{3}{8}\,(2r) = \frac{3}{4}r \Rightarrow r = \frac{4}{3}h$
$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \left(\frac{4}{3}h\right)^2 \cdot h$
$V = \frac{16}{27}\pi h^3 \Rightarrow \frac{dV}{dt} = \frac{16}{9}\pi h^2 \frac{dh}{dt}$

(a) Given: $\frac{dV}{dt} = 10\text{m}^3/\text{min.}$, find $\frac{dh}{dt}$ when $h = 6$.
$10 = \frac{16}{9}\pi\,(6)^2 \cdot \frac{dh}{dt} \Rightarrow 10 = 64\pi \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{5}{32\pi}\text{m/min}$

(b) Find $\frac{dr}{dt}$ at the same time.
$r = \frac{4}{3}h \Rightarrow \frac{dr}{dt} = \frac{4}{3}\frac{dh}{dt} \Rightarrow \frac{dr}{dt} = \frac{5}{24\pi}\text{m/min}$

10. (a) $f(x) = 5\cos(3x) \Rightarrow f'(x) = 5 \cdot (-\sin(3x)) \cdot 3 = -15\sin(3x)$

(b) $g(x) = \sqrt{\sin^2 x} = (\sin^2 x)^{1/2} \Rightarrow g'(x) = \frac{1}{2}(\sin^2 x)^{-1/2} \cdot 2\sin x \cdot \cos x = \frac{\sin x \cos x}{\sqrt{\sin^2 x}}$

11. $x^2 + 3y^2 = 28 \Rightarrow 2x + 6y\frac{dy}{dx} = 0$
at $(4, 2)$: $2(4) + 6(2)\frac{dy}{dx} = 0 \Rightarrow 8 + 12\frac{dy}{dx} = 0$
$m_{tan} = \frac{dy}{dx} = \frac{-8}{12} = -\frac{2}{3} \Rightarrow$ tangent line: $y - 2 = -\frac{2}{3}(x - 4)$

12. Let $a =$ distance of suspect west of intersection
$b =$ distance of police south of intersection
$c =$ distance between police and suspect
so, $a^2 + b^2 = c^2$

We want to find $\frac{dc}{dt}$ when $b = 200$
When $b = 200$, $t = 1\,\text{sec}$, so $a = 80$
$c = \sqrt{80^2 + 200^2} = 40\sqrt{29}$
Also, we know that $\frac{da}{dt} = 80$ and $\frac{db}{dt} = -100$.

Now, $a^2 + b^2 = c^2 \Rightarrow 2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$
$\Rightarrow a\frac{da}{dt} + b\frac{db}{dt} = c\frac{dc}{dt}$
$\Rightarrow 80(80) + 200(-100) = 40\sqrt{29}\frac{dc}{dt} \Rightarrow 160 - 500 = \sqrt{29}\frac{dc}{dt}$
$\Rightarrow \frac{dc}{dt} = -\frac{340}{\sqrt{29}}$

The cars are getting closer together at $-\frac{340}{\sqrt{29}}$ft./s.

13. $y = \frac{1}{x} = x^{-1}$
$\frac{dy}{dx} = -x^{-2}$
at $x = 2$, we have $\frac{dy}{dx} = -(2)^{-2} = -\frac{1}{4}$

14. Let $f(x) = x^3$.
Then $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim h \to 0 \frac{(x+h)^3-x^3}{h}$
$= \lim_{h\to 0} \frac{x^3+3x^2h+3xh^2+h^3-x^3}{h} = \lim h \to 0 \frac{3x^2h+3xh^2+h^3}{h}$
$\lim_{h\to 0} \frac{h(3x^2+3xh+h^2)}{h} = \lim h \to 0 3x^2 + 3xh + h^2$
$= 3x^2$

15.(a) $y = \sqrt{x} - 7x^{-1/3} + \pi$
$y' = \frac{1}{2}x^{-1/2} - 7\left(-\frac{1}{3}\right)x^{-4/3} + 0$
$= \frac{1}{2\sqrt{x}} + \frac{7}{3x^{4/3}}$

(b) $y = (x^2+1)(x^3-2) = x^5 + x^3 - 2x^2 - 2$
$y' = 5x^4 + 3x^2 - 4x$

(c) $y = \frac{\sin x}{\tan x} = \cos x$ (in the domain)
$y' = -\sin x$

16. $y = x^3 - 6x^2 - 15x + 1$
$y' = 3x^2 - 12x - 15 = 3(x^2 - 4x - 5) = 3(x-5)(x+1)$
horizontal tangents at: $x = 5, x = -1$

17. A function can be continuous at $x = 0$, but not have a derivative at that point. A simple example is $y = |x|$; a more complicated example is $y = x\sin\left(\frac{1}{x}\right)$.

Chapter 4 Sample Problems

1. Abolute Maximum: $(0, 1)$; Absolute Minimum: $(1, \sqrt{2} - 2)$.

2. If there were a positive root, call it a then Rolle's Theorem says there must be a critical point between 0 and a. However the derivative is $4x^3 + 15x^2 + 4$ which is always greater than 4 when $x \geq 0$. Thus, no positive root can exist.

3. d.

4. $c = \sqrt{18} - 2$.

5. c.

6. a.) All real $x \neq 0$; b.) & c.) There is a local maximum at $x = -2(2^{2/3})3^{1/3}$, increasing on $(-\infty, -2(2^{2/3})3^{1/3})$ and $(0, \infty)$, decreasing on $(-2(2^{2/3})3^{1/3}, 0)$; d.) & e.) There are no inflection points the function is always concave down.

7. The box should have a square base with sides of length 18, a height of 36 for a maximal volume of 11664.

8. The fence should be $20\sqrt{5}$ by $\dfrac{50}{\sqrt{5}}$.

9. The point $(2^{1/4}, 2^{1/4} + 2^{3/4})$ is closest to the origin.

10. The limit is 3.

11. $a = -4$, $b = 11$.

12. Inflection points occur at $\dfrac{\pm\pi}{2} + 2\pi n$, $\dfrac{\pi}{6} + 2\pi n$, and $\dfrac{5\pi}{6} + 2\pi n$ where n is an integer.

13. a.

14. $t^3 + \sin t + 12$.

15. a) f is increasing on $(-\infty, 0) \cup (4, \infty)$ and decreasing on $(0, 4)$.

b) f is concave up on $(1, \infty)$ and concave down on $(-\infty, 0) \cup (0, 1)$.

16. b) is correct since the derivative of right side of the equation equals to $x \cos 2x$.

17. $x = 2 + \sqrt{2}$ minimize the profit. (Hint: Let $p(x)$ be the profit from selling x items. In other words, $p(x) = r(x) - c(x)$.)

Chapter 5 Sample Problems

1.

a) 1/2 b) $\frac{-1}{3(x^3+1)} + C$

c) 8π (Hint: This is the area of a semi-circle whose radius is 4) d) 212/5

e) $sin 3x/3 + C$ f) $2/3 \cdot (x-4)^{\frac{3}{2}} + 8\sqrt{x-4} + C$ g) 2

2. 24

3. There is not enough information to compute this integral.

4. a) 21 b) $\sum_{i=1}^{n}(3(1+\frac{3i}{n})-2)\cdot\frac{3}{n}$ c) 16.5.

5. a) 36 b) 9 c) True. If it traveled at a rate og 9 for a time og 4, it would have gone 36 units.

6. $2/\pi \approx 0.64$Km

7. See page 283 in the text book.

8. a) 0 b) 30 c) $A(x) = x^2 + 3x - 10$ d) $A'(x) = 2x + 3 = f(x)$

9. 11

10. 1/2. The grid points will be $x_0 = 1$,$x_1 = 1.5$, $x_2 = 2$, $x_3 = 2.5$, $x_4 = 3$. Left points $1, 1.5, 2$, and 2.5. Right points $1.5, 2, 2.5$, and 3. Midpoint $1.25, 1.75, 2.25$, and 2.75.

11. 85.

12. a) 4 , b) $\frac{1}{4} + \frac{\pi}{8}$

13. a) 1/2 , b) $\frac{2}{\pi}$

14. 2

15. $\int_0^7 (x^3 + 4x - 1)\,dx$

16. a) $\sin\sqrt{x}$ b) $\frac{-\sqrt{x+1}}{2\sqrt{x}}$

17. See page 240 in the textbook.

18. See page 269 in the textbook.

19. $3x^2\sin(3x^3 + \sqrt{x^3})$

Chapter 6 Sample Problems

1. $\frac{9}{2}$

2. 6

3. $2 - \sqrt{2}$

4. $\frac{1}{384}(-49 - 33\sqrt{3}3) + \frac{1}{6}(3\sqrt{3} - 4)$

5. $\frac{125}{6}$

6. $\frac{68\pi}{3}$

7. $\frac{3\pi^2}{2} + 4\pi - 2$

8. $\frac{32\pi}{5}$

9. $\frac{32\pi}{3}$

10. π

11. $\frac{4}{9}(\frac{-2+2\sqrt{10^3}}{3})$

12. $L = \int_0^3 \sqrt{1 + 4x^2}\, dx$

13. $25J$